

2019

Three essays on foreign exchange rates

Pichitra Prapassornmanu
Iowa State University

Follow this and additional works at: <https://lib.dr.iastate.edu/etd>

 Part of the [Economics Commons](#)

Recommended Citation

Prapassornmanu, Pichitra, "Three essays on foreign exchange rates" (2019). *Graduate Theses and Dissertations*. 17077.
<https://lib.dr.iastate.edu/etd/17077>

This Dissertation is brought to you for free and open access by the Iowa State University Capstones, Theses and Dissertations at Iowa State University Digital Repository. It has been accepted for inclusion in Graduate Theses and Dissertations by an authorized administrator of Iowa State University Digital Repository. For more information, please contact digirep@iastate.edu.

Three essays on foreign exchange rates

by

Pichitra Prapassornmanu

A dissertation submitted to the graduate faculty
in partial fulfillment of the requirements for the degree of

DOCTOR OF PHILOSOPHY

Major: Economics

Program of Study Committee:
Rajesh Singh, Major Professor
Sergio H. Lence
Joydeep Bhattacharya
Oleksandr Zhylyevskyy
Gary Lyn

The student author, whose presentation of the scholarship herein was approved by the program of study committee, is solely responsible for the content of this dissertation. The Graduate College will ensure this dissertation is globally accessible and will not permit alterations after a degree is conferred.

Iowa State University

Ames, Iowa

2019

Copyright © Pichitra Prapassornmanu, 2019. All rights reserved.

TABLE OF CONTENTS

	Page
LIST OF TABLES	v
LIST OF FIGURES	vi
ACKNOWLEDGEMENTS	ix
ABSTRACT	x
 CHAPTER 1. HETEROGENEOUS INFORMATIONAL HORIZONS AND THE EXCHANGE	
RATE DISCONNECT PUZZLE	1
1.1 Introduction	1
1.2 Information Heterogeneity and the Exchange Rate Disconnect Puzzle	3
1.3 The Model	7
1.4 Simulation Results	16
1.4.1 The Coefficients of the Equilibrium Exchange Rate	16
1.4.2 The Magnification Factor	16
1.4.3 The Effect of Fundamental Shocks on Exchange Rates	20
1.5 Exchange Rate Disconnect Puzzle in Heterogeneous Informational Horizons Framework	20
1.5.1 The Effect of Degree of Persistence	25
1.5.2 The Effect of Private Signal Precision	26
1.6 A Revisit of Information Structure	27
1.6.1 Revised Structure	27
1.6.2 The Role of Heterogeneous Informational Horizons and the Exchange Rate Disconnect with Current Observable Fundamental	32
1.7 Exchange Rate and Order Flow	35
1.7.1 The Structure of the Foreign Exchange Market	35

1.7.2	Understanding Order Flow	36
1.7.3	Demand for Foreign Bond and Order Flow	38
1.7.4	Relationship between Exchange Rate and Order Flow	42
1.8	Conclusion	44
1.9	References	45
CHAPTER 2. REAL EXCHANGE RATES, EXTERNAL IMBALANCES, AND CURRENCY		
	RISK PREMIA	47
2.1	Introduction	47
2.2	The Link between the Real Exchange Rates and External Imbalances, and the UIP Puzzle	51
2.2.1	The UIP Puzzle	51
2.2.2	Currency Risk Premia and Real Exchange Rates	53
2.2.3	Currency Risk Premia and External Imbalances	54
2.3	The Model	56
2.3.1	The Theoretical Model	57
2.3.2	The Estimation Model	64
2.4	Empirical Estimation	64
2.4.1	Data Analysis	64
2.4.2	Estimation Method	68
2.4.3	Estimation Results	70
2.4.4	Forecasting	73
2.5	Conclusion	78
2.6	References	78
CHAPTER 3. EXCHANGE RATE AND UNCOVERED INTEREST RATE PARITY PUZ-		
	ZLE IN A LONG-RUN RISK MODEL	81
3.1	Introduction	81
3.2	Related Literature	85

3.2.1	Long-Run Risk Literature	85
3.2.2	Monetary Policy Represented by Interest Rate Rules	90
3.2.3	Stochastic Discount Factor, Exchange Rate and Uncovered Interest Rate Parity	92
3.3	Model Specification	94
3.3.1	The Model	95
3.3.2	Solving the Model	98
3.3.3	Model Solutions of Exchange Rate and Currency Risk Premium	101
3.4	Quantitative Results	102
3.4.1	Calibration of Parameter Values	103
3.4.2	Simulated Moments of the Model	105
3.4.3	Responses to Shocks	108
3.5	Conclusion	111
3.6	References	112
APPENDIX A. CHAPTER 1 APPENDIX		123
A.1	Higher-Order Iterated Expectation	123
A.2	Invertibility of b_t	124
A.3	Signal Extraction	126
A.4	Expectation of ε_{t+1}^f and ε_{t+2}^f of Group i and j	128
A.5	Expectation of ε_{t-1}^b and ε_t^b of Group i and j	131
APPENDIX B. CHAPTER 2 APPENDIX		133
B.1	Log-Linearized System	133
B.2	UIP Condition	138
APPENDIX C. CHAPTER 3 APPENDIX		139
C.1	The Real Stochastic Discount Factor or the Real Pricing Kernel	139
C.2	The Coefficients	141

LIST OF TABLES

	Page
Table 2.1	Summary Statistics 69
Table 2.2	Estimation Results 72
Table 2.3	Long Horizon Prediction 73
Table 2.4	Testing Historical Averages Against Models 1–4 75
Table 2.5	Testing Model 1 Against Models 2–4 76
Table 2.6	Testing Model 2 Against Model 4 77
Table 2.7	Testing Model 3 Against Model 4 77
Table 3.1	Calibrated Real Parameter Values 105
Table 3.2	Taylor Rules Parameters and Matching Moments 106
Table 3.3	Value of Calibrated Nominal Parameters 107
Table 3.4	Real Variables 108
Table 3.5	Nominal Variables (Basic Rule) 109
Table 3.6	Nominal Variables (Smoothing Interest Rate Rule) 115
Table 3.7	Nominal Variables (Exchange Rate Rule) 116

LIST OF FIGURES

	Page
Figure 1.1 Coefficients, $\rho_b = \rho_f = 0$	17
Figure 1.2 Coefficients, $\rho_b = 0.8 \rho_f = 0$	17
Figure 1.3 Coefficients, $\rho_b = \rho_f = 0.8$	17
Figure 1.4 Coefficients, $\rho_b = 0.8 \rho_f = 1$	17
Figure 1.5 Magnification Factor, $\rho_b = \rho_f = 0$	19
Figure 1.6 Magnification Factor, $\rho_b = 0.8 \rho_f = 0$	19
Figure 1.7 Magnification Factor, $\rho_b = 0 \rho_f = 0.8$	19
Figure 1.8 Magnification Factor, $\rho_b = \rho_f = 0.8$	19
Figure 1.9 F-Shock, $\rho_b = \rho_f = 0$	21
Figure 1.10 B-Shock, $\rho_b = \rho_f = 0$	21
Figure 1.11 F-Shock, $\rho_b = \rho_f = 0.8$	21
Figure 1.12 B-Shock, $\rho_b = \rho_f = 0.8$	21
Figure 1.13 F-Shock, $\rho_b = 0.8 \rho_f = 1$	22
Figure 1.14 B-Shock, $\rho_b = 0.8 \rho_f = 1$	22
Figure 1.15 R^2 , $\rho_b = \rho_f = 0$	24
Figure 1.16 R^2 , $\rho_b = 0.8 \rho_f = 0$	24
Figure 1.17 R^2 , $\rho_b = 0 \rho_f = 0.8$	24
Figure 1.18 R^2 , $\rho_b = \rho_f = 0.8$	24
Figure 1.19 Revised Coefficients, $\rho_b = \rho_f = 0$	30
Figure 1.20 Revised Coefficients, $\rho_b = 0.8 \rho_f = 0$	30
Figure 1.21 Revised Coefficients, $\rho_b = \rho_f = 0.8$	30
Figure 1.22 Revised Coefficients, $\rho_b = 0.8 \rho_f = 1$	30

Figure 1.23	Revised Magnification Factor, $\rho_b = \rho_f = 0$	31
Figure 1.24	Revised Magnification Factor, $\rho_b = 0.8 \rho_f = 0$	31
Figure 1.25	Revised Magnification Factor, $\rho_b = 0 \rho_f = 0.8$	31
Figure 1.26	Revised Magnification Factor, $\rho_b = \rho_f = 0.8$	31
Figure 1.27	Revised R^2 , $\rho_b = \rho_f = 0$	32
Figure 1.28	Revised R^2 , $\rho_b = 0.8 \rho_f = 0$	32
Figure 1.29	Revised R^2 , $\rho_b = 0 \rho_f = 0.8$	33
Figure 1.30	Revised R^2 , $\rho_b = \rho_f = 0.8$	33
Figure 1.31	R^2 in Heterogeneous Information Framework	34
Figure 1.32	F-Shock on $B_{F,t}^i$	40
Figure 1.33	F-Shock on $B_{F,t}^j$	40
Figure 1.34	B-Shock on $B_{F,t}^i$	41
Figure 1.35	B-Shock on $B_{F,t}^j$	41
Figure 1.36	F-Shock on Aggregate Order Flow	42
Figure 1.37	B-Shock on Aggregate Order Flow	42
Figure 1.38	R^2 on Order Flow, $\rho_b = \rho_f = 0$	43
Figure 1.39	R^2 on Order Flow, $\rho_b = 0.8 \rho_f = 0$	43
Figure 1.40	R^2 on Order Flow, $\rho_b = 0 \rho_f = 0.8$	44
Figure 1.41	R^2 on Order Flow, $\rho_b = \rho_f = 0.8$	44
Figure 2.1	Australia	67
Figure 2.2	Canada	67
Figure 2.3	Japan	67
Figure 2.4	New Zealand	67
Figure 2.5	Norway	67
Figure 2.6	Switzerland	67
Figure 2.7	United Kingdom	68
Figure 2.8	Unites States	68

Figure 3.1	Dynamic Responses to the Short-Run Consumption Level Shock, ε_t^x	117
Figure 3.2	Dynamic Responses to the Long-Run Consumption Level Shock, ε_t^z	118
Figure 3.3	Dynamic Responses to the Monetary Policy Level Shock, ε_t^v	119
Figure 3.4	Dynamic Responses to the Short-Run Consumption Volatility Shock, ε_t^u . .	120
Figure 3.5	Dynamic Responses to the Long-Run Consumption Volatility Shock, ε_t^w . .	121
Figure 3.6	Dynamic Responses to the Monetary Policy Volatility Shock, ε_t^e	122

ACKNOWLEDGEMENTS

I would like to express the deepest gratitude to my major professor, Dr. Rajesh Singh, for his guidance, expertise, strong support, and most importantly, for his trust throughout the writing of this dissertation. I feel very lucky to have a chance working under his supervising. I would also express my big thanks to my committee members: Dr. Sergio H. Lence, Dr. Joydeep Bhattacharya, Dr. Oleksandr Zhylyevskyy, Dr. Gary Lyn, and Dr. Harvey Lapan, who give me valuable suggestions and advises on this dissertation. My thanks would also go to people in the macroeconomics reading group, leading by Dr. Juan Carlos Cordoba, for their constructive comments that improve this dissertation.

I immensely thank Thammasat University for granting a scholarship throughout my Ph.D study. My thanks would go to Mrs. Jeri Prescott for honoring me the 2017 James R. Prescott Scholarship in Economics. This award inspires and pushes me further to conduct my research. I also would like to specially thank Dr. John Beghin, Dr. Bruce Babcock, and Dr. Sebastien Pouliot for giving me a chance to work as their research assistant. This opportunity helps accumulating my research skill.

I am indebted to my family for their love, encouragement, and always believing in me. I am deeply grateful to my husband for his love, support, and sacrifice. I would like to thank him with all my heart for always being there for me. Last but not least, I would like to thank my friends, colleagues, the department faculty and staff for their support and assistance. I would not finish writing this dissertation and would not have been to this point without help and encouragement from all of these people.

ABSTRACT

This dissertation focuses on the behavior of the exchange rate and the currency risk premium. The first chapter studies the problem of exchange rate disconnect from economic fundamentals by analyzing the role of heterogeneous information among investors. The second paper examines the relationship between currency risk premia, interest rate differentials, real exchange rates, and external imbalances. The third chapter investigates the violation of uncovered interest rate parity, the exchange rate, and the currency risk premium in a model where consumption growth prospects contain a long-run risk component with the stochastic volatility.

CHAPTER 1. HETEROGENEOUS INFORMATIONAL HORIZONS AND THE EXCHANGE RATE DISCONNECT PUZZLE

In this chapter, I study the problem of exchange rate “disconnect” from fundamentals in a model where investors have heterogeneous informational horizons about future fundamentals. The disconnect between the exchange rate and the observable economic fundamental arises because of the existence of private information across investors. The model has two types of investors. A fraction of these investors are better informed in the sense that they are able to obtain information about future fundamentals in advance of the remaining investors with more precise private signal quality. There are two main findings. First, the disconnect problem is serious when a group of investors holds different beliefs about future fundamentals, and it is even worse when they receive information about these fundamentals at a longer period. In fact, as the fraction of better-informed investors increases, the disconnect problem aggravates. Second, as the fundamental is more persistent, the exchange rate is more connected with the current observable fundamental. Furthermore, if one studies the relationship between exchange rate and economic fundamentals, one should not miss out the role of private information which can be captured by order flow. Numerical analysis of the model exhibits a close relationship between exchange rate and order flow.

1.1 Introduction

The mainstream macroeconomic models of exchange rate determination, for example, the flexible price monetary model and the sticky price monetary model, mainly rely on public macroeconomic information. These traditional models assume rational expectations under the assumption that information is publicly shared by all agents. There is no role of private information. In these models, exchange rates are determined by macroeconomic fundamentals, namely, interest rate, money supply, inflation, and income. However, these models fail to explain exchange rate

dynamics, especially at a short horizon. The weak relationship between the exchange rate and its macroeconomic fundamental is called the exchange rate disconnect puzzle. Meese and Rogoff (1983) conclude that a random walk model predicts exchange rates better than the models based on macroeconomic models. Subsequent research, e.g., Mark (1995), found a better prediction of exchange rate by fundamentals at a longer horizon.

One possible reason for this poor relationship is that traditional models do not take into account the mechanics of the foreign exchange market, especially the role of dealers, and therefore miss out on the role of private information and agent heterogeneity. In contrast, the market microstructure approach incorporates the features of trading in the foreign exchange market into a model. This approach has an advantage over its traditional counterparts because it links the information relevant to macroeconomic fundamentals with the spot exchange rate through the trading process. More specifically, research on foreign exchange market microstructure focuses on the role of private information, and the heterogeneity of agents in determining exchange rates. By incorporating these features, microstructure models have the potential to successfully explain the exchange rate disconnect puzzle.

Bacchetta and van Wincoop (2006) introduce heterogeneous information into a dynamic monetary model of exchange rate determination. In their model, a continuum of investors receive symmetrically dispersed information about future macroeconomic fundamentals. I study the role of information heterogeneity in explaining the exchange rate disconnect puzzle by extending their model in the sense that a group of investors received asymmetrically private signals about future fundamentals. In my model, investors receive private signals about future fundamentals at different horizons. This would cause an informational advantage to the group of investors who receive signals about future fundamentals of more periods ahead. The disconnect problem is strong when investors receive information at different periods ahead. And, an increase in the fraction of investors who know information at a long period intensifies the problem. When there are more investors who know information about future fundamentals at a long period ahead, the importance of the heterogeneity among private information is greater, and hence the unobserved components

in the equilibrium exchange rate become more evident. The relationship between the exchange rate and current observable fundamental deteriorates. However, when the process of the observable fundamental is persistent, the future fundamental highly correlates with the current one. An increase in the fraction of investors who know more about future fundamentals would help the exchange rate connect with the current observable fundamental. Therefore, knowing more about future fundamentals would not help the exchange rate to connect with the current observable fundamental unless the process of fundamentals f_t is more persistent. Moreover, the model with heterogeneous informational horizon also exhibits the implication of private information through order flow. I find that the observed fundamental has little explanatory power on the exchange rate. Instead, the exchange rate is firmly related to order flow.

The rest of the chapter is organized as follows. Section 1.2 presents the idea of the effect of private information on the exchange rate. Section 1.3 lays out the details of the model. The model implications along with numerical results are also discussed in section 1.4. Section 1.5 provides an analysis of the disconnect puzzle. In section 1.6, I develop another setting of information structure that causes an informational advantage to one group of investors. In section 1.7, I present the role of order flow on the exchange rate. Last but not least, section 1.8 provides some concluding remarks.

1.2 Information Heterogeneity and the Exchange Rate Disconnect Puzzle

Information heterogeneity is key to understand dynamic in exchange rate. Rather than attempting to connect macroeconomic variables to the exchange rate, the market-microstructure approach addresses the importance of the trading foreign exchange rate process that impounds macro information on the exchange rate. As a matter of fact, not all relevant macro information is publicly known to investors. There exists information that is being dispersed among agents in the economy. The spot exchange rate reflects this heterogeneous information that is available to the dealers because it is the price of foreign currency quoted by dealers.

One might question the existence of private information in currency market because in the traditional macro models most economic fundamentals that link to exchange rates are believed to be publicly known to all agents in the market. However, macroeconomic information is available in a limited amount to agents in the economy. Private information exists in the market for various reasons. For example, investors may receive different information about future fundamentals because they have different timing gaps when acquiring the public information. Some investors may have better foresight about future fundamentals than others in the economy. Like in this paper, one group of investors can infer about future fundamentals at a longer period ahead the other group. Therefore, information about future economic fundamentals, though publicly announced, can be dispersed among investors and dealers. Trading in the foreign exchange market is based on the access to different information sets. Each agent's trade in the market depends on his own information and perception reaching to him. The dispersed information inevitably affects sales and orders for the foreign currency of individual investor.

The market microstructure literature posits that market participants have different sets of information. Dispersed information among agents critically impacts exchange rate dynamics. Bacchetta and van Wincoop (2006) address the exchange rate determination puzzle by introducing heterogeneous information into a standard monetary model of exchange rate determination. In their model, there are two types of fundamentals, namely, f fundamental and b fundamental. A continuum of investors differ in two respects. First, they observe f_t and receive symmetrically dispersed information about future f fundamentals. Second, they all have different exchange rate risk exposure associated with their non-asset income. This exposure is private information and it leads to hedge trades whose aggregation is also unobservable. The unobservable hedge trade is referred to b fundamental. The paper's key result is that information heterogeneity disconnects the exchange rate from fundamentals in the short run but allows a close relationship in the long run. In the short run, investors do not know whether the change in the exchange rate is driven by the average private signals about future fundamentals or the unobserved shock. They called this effect as "rational confusion". There is also a magnification effect on the unobserved component. The impact of an

unobserved shock on exchange rates is big and persistent in the heterogeneous model. Therefore, in the short run, unobserved fundamentals dominate exchange rate volatility. Rational confusion and the persistence of unobserved shock diminish as investors learn more about future fundamentals. The impact of the unobserved fundamentals on the exchange rate weakens in the long run. This results in a close relationship between the exchange rate and observed fundamentals in the long run.

In this study, I add a novel and a realistic dimension to Bacchetta and van Wincoop's model with information heterogeneity among investors. In particular, I introduce differences in informational horizons across investors. In my model, there is one group of investors that receives information about future fundamentals in advance of another group. Equivalently, there is an information lag between two groups of investors. I find that the disconnect problem is strengthened when a group of investors receive a relatively longer horizon signal in advance.

Bacchetta and van Wincoop's model generates an equilibrium exchange rate that depends on an average of higher order expectations of current and future economic fundamentals, composed of observed and unobserved parts. Private information takes the form of a signal about future fundamentals. At time t , investors observe all past and current fundamentals. They receive a private signal each period about the observable fundamental T periods ahead. Each investor infers his expectation of future fundamentals from the distribution of future fundamentals, the adjusted exchange rate, and a signal about future fundamentals. Then the average expectation is computed by aggregating each investor's expectation.

For the simplest case where $T = 1$, the equilibrium exchange rate depends on an observed current fundamental, a fundamental at the next period, and a current unobserved fundamental. They compare the R^2 of the model with private information and the model with common knowledge. The result is that the R^2 is lower in the heterogeneous information model, suggesting that with private information the exchange rate disconnects from the observed fundamental.

There are two dynamic properties of the model when $T > 1$. First, the impact of unobservable shocks on the exchange rate are persistent. Second, expectations of infinite order of fundamentals

affect the exchange rate. When the unobservable shocks are persistent, it is more difficult for investors to learn about fundamentals up to time $t + T$ from exchange rates at time t . Hence, deviations of the exchange rate from observed fundamentals can be very long-lasting. With the presence of higher-order expectations, the unobserved hedge trades receive a larger weight in the equilibrium exchange rate. On the other hand, the exchange rate depends less on unobserved future fundamentals. Therefore, the overall effect on the connection between the exchange rate and observed fundamentals is ambiguous.

Whereas in Bacchetta and van Wincoop's model all investors receive symmetric information dispersion about future fundamentals, in my study the information structure is asymmetric across groups of investors. Specifically, one group of investors receives private signals about observable fundamentals two periods ahead, while the other group receives only one-period ahead signals. This model set-up is intended to replicate the observed market feature that investors are differentially informed. I study three important scenarios. In scenario 1, all investors receive one-period ahead private signals about future fundamentals. They can extract information only about future fundamentals one period ahead. In scenario 2, there is one group of investors receiving two-period ahead private signals and another group receives one-period ahead signals. An informational advantage accrues to the investors who know the two-period ahead signals because they can directly extract information about one- and two-period ahead of future fundamentals from the number of signals greater than the other group who only receive one-period ahead signals. Though a group of one-period signal investors does not receive signal about two-period ahead fundamental, they make an inference about this fundamental through the exchange rate signal. In scenario 3, all investors receive two-period ahead signals about future fundamentals. They all extract information up to two-period ahead about future fundamentals.

I find that the disconnect problem is strong when there is a group of investors who are better informed about future fundamentals. The better-informed investors are the investors who are able to obtain information about future fundamentals in advance of the other investors with more precise private signal quality. Even if they hold information about future fundamentals at a longer period,

the disconnect problem is more intense. As the fraction of better-informed investors increases, exchange rates are more disconnected with the current observable fundamental. The reason is that an increase in the number of better-informed investors increases an existence of the unobservable components, e.g. future fundamentals, in the model. This would bring down the R^2 in the regression of the exchange rate on the current observable fundamental. Moreover, the degree of fundamentals persistence plays a vital role in determining the disconnect problem. When the fundamental is more persistent, the R^2 increases. As the fundamental is more persistent, the future fundamentals would highly correlate with the current fundamental. Knowing more about future fundamentals would help the exchange rate correlate with the current fundamental.

1.3 The Model

This model modifies the standard monetary model of exchange rate determination by introducing a heterogeneous informational horizon of private signal about future fundamentals into the model. There is a continuum of investors who are divided into two groups. Each group receives asymmetrically dispersed information about future fundamentals. Information asymmetry is caused by the information lag in which one group of investors receives a signal about future fundamental one period ahead of the other group. Under heterogeneous information, the exchange rate will depend on the higher-order expectation of future fundamentals.

There are two economies: Home and Foreign countries, that produce the same good so that purchasing power parity holds

$$p_t = p_t^* + s_t, \quad (1.1)$$

where $p_t = \ln(P_t)$, $p_t^* = \ln(P_t^*)$, and $s_t = \ln(S_t)$, where P_t and P_t^* are the local and foreign currency prices, and S_t is the nominal exchange rate (home currency per unit of foreign currency).

The basic setup closely follows the monetary model with information dispersion presented by Bacchetta and van Wincoop (2006). There is a continuum of investors in both countries on the interval $[0, 1]$. An investor lives for two periods. There are overlapping generations in the sense that, before dying, investor $i \in [0, 1]$ passes on his private information to the next investor born

in the following period. So at time t , investor i observes all lagged and current values of the fundamentals. Each investor i makes only one investment decision and he chooses to invest in four assets: money of his own country, nominal bonds of both countries (b_{Ht}^i, b_{Ft}^i) with interest rates i_t and i_t^* , and a technology with fixed real return (r). I assume a small open-economy setting in which Home country is large and Foreign country is very small. Bond market equilibrium is therefore entirely determined by investors in the large Home country. Money supply in the Home country is constant, implying a constant price level p_t in equilibrium, so that $i_t = r$. Money supply in the Foreign country is stochastic.

Investors born at time t are endowed with w_t^i . At time $t + 1$, investors receive the return from investment plus income y_{t+1}^i from production at time $t + 1$. Income depends on both the exchange rate and the real money holding, denoted by \tilde{m}_t^i , through the function $y_{t+1}^i = \lambda_t^i s_{t+1} - \tilde{m}_t^i (\ln(\tilde{m}_t^i) - 1)/\alpha$, with $\alpha > 0$. The benefit of having money in the production rather than utility is to avoid making money demand a function of consumption which complicates the solution. Note that λ_t^i is the degree of exchange rate exposure to the nonasset income of investor i . It is time varying and known only to investor i . The budget constraint of agent i at time t is

$$w_t^i = \tilde{m}_t^i + b_{Ht}^i + S_t b_{Ft}^i,$$

whereas the budget constraint of agent i at time $t + 1$ is

$$c_{t+1}^i = \tilde{m}_t^i + (1 + i_t) b_{Ht}^i + (1 + i_t^*) S_{t+1} b_{Ft}^i + y_{t+1}^i.$$

Combining the two yields:

$$c_{t+1}^i = (1 + i_t) w_t^i - i_t \tilde{m}_t^i + [(1 + i_t^*) S_{t+1} - (1 + i_t) S_t] b_{Ft}^i + y_{t+1}^i.$$

The term in the bracket, $(1 + i_t^*) S_{t+1} - (1 + i_t) S_t$, can be approximated as $i_t^* + s_{t+1} - i_t - s_t$, using the log-linearization that $\ln(1 + x) \approx x$ and $\ln(z) \approx z - 1$. The term $s_{t+1} - s_t + i_t^* - i_t$ is called the log-linearized excess return on investing abroad. Therefore, the budget constraint can be rewritten as:

$$c_{t+1}^i = (1 + i_t) w_t^i - i_t \tilde{m}_t^i + (s_{t+1} - s_t + i_t^* - i_t) b_{Ft}^i + y_{t+1}^i. \quad (1.2)$$

Investor i maximizes his expected CARA utility, denoted by $-E_t^i e^{-\gamma c_{t+1}^i}$, subject to (1.2). Assuming that, conditional on the information at time t of agent i , I_t^i , $s_{t+1}|I_t^i \sim N(E_t^i(s_{t+1}), \sigma_t^2)$. The first-order conditions with respect to money holding and investment in foreign bonds are, respectively:

$$\begin{aligned} \ln(\tilde{m}_t^i) + \alpha i_t &= 0, \\ E_t^i(s_{t+1} - s_t + i_t^* - i_t) - \gamma \sigma_t^2 (b_{Ft}^i + \lambda_t^i) &= 0. \end{aligned} \quad (1.3)$$

Combining the first-order condition for money holding with money market equilibrium in both countries gets

$$m_t - p_t = -\alpha i_t, \quad (1.4)$$

$$m_t^* - p_t^* = -\alpha i_t^* \quad (1.5)$$

where m_t and m_t^* are the logs of domestic and foreign nominal money supply.

The nonasset income y_{t+1}^i depends on λ_t^i , which represents the exchange rate exposure by investor i . Let b_t^i be the hedge against nonasset income of an investor i . This implies that investor i needs to hedge by the amount $b_t^i = \frac{\partial y_{t+1}^i}{\partial s_{t+1}} = \lambda_t^i$. Hence, the hedge against non-asset income adds to the demand for the foreign bond.

From equation (1.3), the demand for the foreign bond can be written as

$$b_{Ft}^i = \frac{E_t^i(s_{t+1}) - s_t + i_t^* - i_t}{\gamma \sigma_t^2} - b_t^i. \quad (1.6)$$

The first term depends on the excess return on investing in foreign bonds and the risk aversion in the denominator. The second term is the hedging demand against exchange rate exposure in the investor's non-asset income.

Hedging demand caused by the exchange rate exposure, denoted by b_t^i , is assumed to be equal to the average term, denoted by b_t , plus an idiosyncratic term:

$$b_t^i = b_t + \varepsilon_t^i.$$

Every investor observes his own exposure term, but this provides no information about the average exposure. That is, the average hedging demand is unobservable to any of the investors. While b_t

is an unobserved fundamental in the model, the assumed autoregressive process is known by all agents. I assumed that b_t follows an AR(1) process

$$b_t = \rho_b b_{t-1} + \varepsilon_t^b,$$

where $\varepsilon_t^b | I_{t-1} \sim N(0, \sigma_b^2)$.

In equilibrium, bonds are in zero net supply, market equilibrium is given by $\int_0^1 b_{Ft}^i di = 0$. Integrating equation (1.6) over all investors yields the interest rate arbitrage condition

$$\bar{E}_t[s_{t+1}] - s_t = i_t - i_t^* + \gamma \sigma_t^2 b_t, \quad (1.7)$$

where \bar{E}_t is the average rational expectation across investors. The risk premium term, denoted by $\gamma \sigma_t^2 b_t$, depends on the coefficient of absolute risk aversion (γ), the conditional variance of the exchange rate (σ_t^2), and the average hedging demand (b_t).

Fundamentals in the model are categorized into f fundamental and b fundamental. The observable f fundamental, denoted by f_t , is the difference of the logs of domestic and foreign nominal money supply ($m_t - m_t^*$). The future f_t 's are unobservable, but investors can infer about them from private signals about future fundamentals they receive. The b fundamental is unobservable and introduces noise that prevents investors from inferring the average expectation of future f fundamentals from the exchange rate. Equations (1.1), (1.4), and (1.5) imply that $-(i_t - i_t^*) = \frac{f_t - s_t}{\alpha}$. From equation (1.7), the equilibrium of exchange rate equation is as follows:

$$s_t = \frac{1}{1 + \alpha} f_t - \frac{\alpha}{1 + \alpha} \gamma \sigma_t^2 b_t + \frac{\alpha}{1 + \alpha} \bar{E}_t[s_{t+1}]. \quad (1.8)$$

Substituting for the average expectation, the equilibrium exchange rate can be written as

$$s_t = \frac{1}{1 + \alpha} \sum_{k=0}^{\infty} \left(\frac{\alpha}{1 + \alpha} \right)^k \bar{E}_t^k [f_{t+k} - \alpha \gamma \sigma_{t+k}^2 b_{t+k}], \quad (1.9)$$

where $E_t^0[x_t] = x_t$, $\bar{E}_t^1[x_{t+1}] = \bar{E}_t[x_{t+1}]$, and higher-order expectation are defined as

$$\bar{E}_t^k[x_{t+k}] = \bar{E}_t \bar{E}_{t+1} \dots \bar{E}_{t+k-1}[x_{t+k}].$$

The law of iterated expectations does not apply to the average expectation, i.e. $\bar{E}_t \bar{E}_{t+1}[s_{t+2}] \neq \bar{E}_t[s_{t+2}]$ under heterogeneous information. I explain this in Appendix A.1. The equilibrium exchange rate equation indicates that the exchange rate at time t depends on the fundamental at

time t , the average expectation at time t of the fundamental at time $t + 1$, and the higher orders of the average expectation of the future fundamentals at times $t + 2, t + 3$, and so on. De Grauwe and Grimaldi (2005) critique the model of heterogeneous agents rational expectation, arguing that it creates an infinite regress in which the exchange rate depends on the expectation of other agents' expectations, which depends on the expectations of the expectations of other agents' expectation and so on. This leads to intractable mathematic problems unless some simplifying assumption are made. However, it is unnecessary to compute all higher-order expectations. The key equation used to find the solution for exchange rate is equation (1.8), which is derived from the interest arbitrage condition. The signal extraction method is used to solve for the average expectation of future fundamentals, and the method of undetermined coefficients is to compute all coefficients satisfying the equilibrium of exchange rate.

Information Structure

In a continuum of investors, there are two groups of them. Each group $\{i, j\}$ receives a different signal about future fundamentals. Let $\omega \in [0, 1]$. There are ω of investors in group i , and $(1 - \omega)$ of investors in group j . The information received by each group differs in the timing of the future fundamentals received. There is one group of investors who are early-informed about future fundamentals in the sense that they receive a signal about future fundamentals one period ahead of the other group.

At time t all investors observe all past and current fundamentals. Each period, investors in group i receive a private signal of the two-period ahead signal about future fundamentals whereas investors in group j receive a one-period ahead signal. That is, investors in group i receive the private signal

$$v_t^i = f_{t+2} + \varepsilon_t^{v_i}, \varepsilon_t^{v_i} \sim N(0, \sigma_{v_i}^2).$$

Investors in group j receive the private signal

$$v_t^j = f_{t+1} + \varepsilon_t^{v_j}, \varepsilon_t^{v_j} \sim N(0, \sigma_{v_j}^2).$$

Group i investors know information about a farther period ahead of group j investors. In other words, they are early-informed investors.

Furthermore, it is assumed that all investors know the process of observable fundamental which is governed by

$$f_t = \rho_f f_{t-1} + \varepsilon_t^f, \varepsilon_t^f \sim N(0, \sigma_f^2).$$

Solving the Model

The model is solved in three steps. First, I conjecture a solution for the exchange rate, which depends on shocks to observable and unobservable fundamentals. Second, I apply signal extraction to compute the average expectation of these shocks, and substitute them into the exchange rate equilibrium equation (1.8). Third, I solve for coefficients in the exchange rate equilibrium equation by the method of undetermined coefficients. Note that at time t , the average exposure b_t and b_{t-1} is unobservable, but b_{t-2} and earlier lags are known if it is assumed that the process of b_t is invertible. The invertibility of b_t is discussed in Appendix A.2.

To begin with, I conjecture a solution that the exchange rate depends on current and future observable fundamentals up to time $t + 2$, and two-period lags of unobservable fundamentals up to the current one as

$$s_t = \lambda_0 f_t + \lambda_1 f_{t+1} + \lambda_2 f_{t+2} + \gamma_{-2} b_{t-2} + \gamma_{-1} b_{t-1} + \gamma_0 b_t.$$

I assume that f_t , and b_t follow an AR process as $f_t = \rho_f f_{t-1} + \varepsilon_t^f$ and $b_t = \rho_b b_{t-1} + \varepsilon_t^b$. Thus, the conjectured equation becomes

$$s_t = \tilde{\lambda}_0 f_t + \tilde{\lambda}_1 \varepsilon_{t+1}^f + \tilde{\lambda}_2 \varepsilon_{t+2}^f + \tilde{\gamma}_{-2} b_{t-2} + \tilde{\gamma}_{-1} \varepsilon_{t-1}^b + \tilde{\gamma}_0 \varepsilon_t^b, \quad (1.10)$$

where $\tilde{\lambda}_0 = \lambda_0 + \lambda_1 \rho_f + \lambda_2 \rho_f^2$, $\tilde{\lambda}_1 = \lambda_1 + \lambda_2 \rho_f$, $\tilde{\lambda}_2 = \lambda_2$, $\tilde{\gamma}_{-2} = \gamma_{-2} + \gamma_{-1} \rho_b + \gamma_0 \rho_b^2$, $\tilde{\gamma}_{-1} = \gamma_{-1} + \gamma_0 \rho_b$, and $\tilde{\gamma}_0 = \gamma_0$.

Next, I will compute the first-order average expectation of ε_{t+1}^f , denoted by $\bar{E}_t \varepsilon_{t+1}^f$, which is a weighted average of the group- i expectation at time t of ε_{t+1}^f , denoted by $E_t^i \varepsilon_{t+1}^f$, and the group- j expectation at time t of ε_{t+1}^f , denoted by $E_t^j \varepsilon_{t+1}^f$. With the same method, I compute $\bar{E}_t \varepsilon_{t+2}^f$. Finally, I substitute these computations into the exchange rate equilibrium equation.

In order to solve the signal extraction problem, I need to gather all signals that provide information about the unknown innovations. Group i investors extract information about ε_{t+1}^f and ε_{t+2}^f from four signals, while group j investors can extract information from three signals. At time t , group i investors not only extract information about ε_{t+1}^f and ε_{t+2}^f from their private signals v_{t-1}^i and v_t^i , but also from the exchange rates s_t and s_{t-1} . Clearly, at time t the private signal v_{t-1}^i is informative about ε_{t+1}^f . The private signal v_t^i is informative about ε_{t+1}^f and ε_{t+2}^f . As such, I can write

$$\begin{aligned} v_t^i - \rho_f^2 f_t &= \rho_f \varepsilon_{t+1}^f + \varepsilon_{t+2}^f + \varepsilon_t^{v_i}, \\ v_{t-1}^i - \rho_f f_t &= \varepsilon_{t+1}^f + \varepsilon_{t-1}^{v_i}. \end{aligned}$$

For group j , the private signal is

$$v_t^j - \rho_f f_t = \varepsilon_{t+1}^f + \varepsilon_t^{v_j}.$$

Since the exchange rate s_t depends on ε_{t+1}^f and ε_{t+2}^f , the exchange rate itself becomes a source of information about these innovations. With the same logic, s_{t-1} is also a source of ε_{t+1}^f information. Define $\tilde{s}_t = s_t - \tilde{\lambda}_0 f_t$. I have two signals from the exchange rate as follows

$$\begin{aligned} \tilde{s}_t - \tilde{\gamma}_{-2} b_{t-2} &= \tilde{\lambda}_1 \varepsilon_{t+1}^f + \tilde{\lambda}_2 \varepsilon_{t+2}^f + \tilde{\gamma}_{-1} \varepsilon_{t-1}^b + \tilde{\gamma}_0 \varepsilon_t^b, \\ \tilde{s}_{t-1} - \tilde{\lambda}_1 \varepsilon_t^f - \tilde{\gamma}_{-1} \varepsilon_{t-2}^b - \tilde{\gamma}_{-2} b_{t-3} &= \tilde{\lambda}_2 \varepsilon_{t+1}^f + \tilde{\gamma}_0 \varepsilon_{t-1}^b. \end{aligned}$$

For signal extraction $\mathbf{x} = \mathbf{A}\boldsymbol{\theta} + \boldsymbol{\varepsilon}$, where \mathbf{x} is a vector of signals, $\boldsymbol{\theta}$ is a vector of information, $\boldsymbol{\varepsilon}$ is a vector of signal errors, $\boldsymbol{\theta} \sim N(\boldsymbol{\mu}, \boldsymbol{\Sigma}_\theta)$ and $\boldsymbol{\varepsilon} \sim N(\mathbf{0}, \boldsymbol{\Sigma}_\varepsilon)$ independent of $\boldsymbol{\theta}$, then $\boldsymbol{\theta}|\mathbf{x} \sim N(\boldsymbol{\mu} + (\boldsymbol{\Sigma}_\theta^{-1} + \mathbf{A}'\boldsymbol{\Sigma}_\varepsilon^{-1}\mathbf{A})^{-1}\mathbf{A}'\boldsymbol{\Sigma}_\varepsilon^{-1}(\mathbf{x} - \mathbf{A}\boldsymbol{\mu}), (\boldsymbol{\Sigma}_\theta^{-1} + \mathbf{A}'\boldsymbol{\Sigma}_\varepsilon^{-1}\mathbf{A})^{-1})$. Let $\boldsymbol{\theta} = (\varepsilon_{t+1}^f, \varepsilon_{t+2}^f)'$, one can compute their conditional expectations. The derivation of $E_t^i \varepsilon_{t+1}^f$, $E_t^i \varepsilon_{t+2}^f$, $E_t^j \varepsilon_{t+1}^f$, and $E_t^j \varepsilon_{t+2}^f$ is shown in Appendix A.3, and they can be written as:

$$\begin{aligned} E_t^i \varepsilon_{t+1}^f &= a_1 v_t^i + a_2 v_{t-1}^i + a_3 s_t + a_4 b_{t-2} + a_5 f_t + a_6 \hat{s}_{t-1}, \\ E_t^i \varepsilon_{t+2}^f &= c_1 v_t^i + c_2 v_{t-1}^i + c_3 s_t + c_4 b_{t-2} + c_5 f_t + c_6 \hat{s}_{t-1}, \\ E_t^j \varepsilon_{t+1}^f &= d_1 v_t^j + d_3 s_t + d_4 b_{t-2} + d_5 f_t + d_6 \hat{s}_{t-1}, \\ E_t^j \varepsilon_{t+2}^f &= e_1 v_t^j + e_3 s_t + e_4 b_{t-2} + e_5 f_t + e_6 \hat{s}_{t-1}. \end{aligned}$$

I integrate the group i and j 's expectations of ε_{t+1}^f and ε_{t+2}^f over the continuum of investors in order to achieve $\bar{E}_t \varepsilon_{t+1}^f$ and $\bar{E}_t \varepsilon_{t+2}^f$. The average expectation of the two shocks are shown in the following equations:

$$\begin{aligned}\bar{E}_t \varepsilon_{t+1}^f &= \{\omega(a_1 \rho_f^2 + a_2 \rho_f + a_5) + (1 - \omega)(d_1 \rho_f + d_5)\} f_t \\ &\quad + \{\omega(a_1 \rho_f + a_2 + a_6 \tilde{\lambda}_2) + (1 - \omega)(d_1 + d_6 \tilde{\lambda}_2)\} \varepsilon_{t+1}^f \\ &\quad + \omega a_1 \varepsilon_{t+2}^f + \{\omega a_3 + (1 - \omega) d_3\} s_t \\ &\quad + \{\omega a_4 + (1 - \omega) d_4\} b_{t-2} + \{\omega a_6 \tilde{\lambda}_0 + (1 - \omega) d_6 \tilde{\lambda}_0\} \varepsilon_{t-1}^b, \\ \bar{E}_t \varepsilon_{t+2}^f &= \{\omega(c_1 \rho_f^2 + c_2 \rho_f + c_5) + (1 - \omega)(e_1 \rho_f + e_5)\} f_t \\ &\quad + \{\omega(c_1 \rho_f + c_2 + c_6 \tilde{\lambda}_2) + (1 - \omega)(e_1 + e_6 \tilde{\lambda}_2)\} \varepsilon_{t+1}^f \\ &\quad + \omega c_1 \varepsilon_{t+2}^f + \{\omega c_3 + (1 - \omega) e_3\} s_t \\ &\quad + \{\omega c_4 + (1 - \omega) e_4\} b_{t-2} + \{\omega c_6 \tilde{\lambda}_0 + (1 - \omega) e_6 \tilde{\lambda}_0\} \varepsilon_{t-1}^b.\end{aligned}$$

By forwarding one period ahead the conjectured exchange rate equation and taking the average expectation on both sides, I obtain

$$\bar{E}_t s_{t+1} = \tilde{\lambda}_0 \rho_f f_t + \tilde{\gamma}_{-2} \rho_b b_{t-2} + \tilde{\lambda}_0 \bar{E}_t \varepsilon_{t+1}^f + \tilde{\lambda}_1 \bar{E}_t \varepsilon_{t+2}^f + \tilde{\gamma}_{-2} \bar{E}_t \varepsilon_{t-1}^b + \tilde{\gamma}_{-1} \bar{E}_t \varepsilon_t^b.$$

From the exchange rate equilibrium equation (1.8), I substitute for $\bar{E}_t s_{t+1}$, and achieve

$$\begin{aligned}s_t &= \frac{1}{1 + \alpha} [1 + \alpha \tilde{\lambda}_0 \rho_f] f_t - \frac{\alpha}{1 + \alpha} [\gamma \sigma_t^2 \rho_b^2 - \tilde{\gamma}_{-2} \rho_b] b_{t-2} - \frac{\alpha}{1 + \alpha} \gamma \sigma_t^2 \rho_b \varepsilon_{t-1}^b - \frac{\alpha}{1 + \alpha} \gamma \sigma_t^2 \varepsilon_t^b \\ &\quad + \frac{\alpha}{1 + \alpha} [\tilde{\lambda}_0 \bar{E}_t \varepsilon_{t+1}^f + \tilde{\lambda}_1 \bar{E}_t \varepsilon_{t+2}^f + \tilde{\gamma}_{-2} \bar{E}_t \varepsilon_{t-1}^b + \tilde{\gamma}_{-1} \bar{E}_t \varepsilon_t^b].\end{aligned}\quad (1.11)$$

Substituting the above equation with $\bar{E}_t \varepsilon_{t+1}^f$, $\bar{E}_t \varepsilon_{t+2}^f$, $\bar{E}_t \varepsilon_{t-1}^b$ and $\bar{E}_t \varepsilon_t^b$,¹ the equilibrium of the exchange rate is shown by

¹The calculations of $\bar{E}_t \varepsilon_{t-1}^b$ and $\bar{E}_t \varepsilon_t^b$ are shown in Appendix A.5

$$\begin{aligned}
s_t = & \frac{1}{1 + \alpha(1 - \rho_f)} f_t - \frac{\alpha \gamma \sigma_t^2 \rho_b^2}{1 + \alpha(1 - \rho_b)} b_{t-2} \\
& + \frac{\alpha}{1 + \alpha} \left[\tilde{\lambda}_0 \{ \omega(a_1 \rho_f + a_2 + a_6 \tilde{\lambda}_2) + (1 - \omega)(d_1 + d_6 \tilde{\lambda}_2) \} \right. \\
& + \tilde{\lambda}_1 \{ \omega(c_1 \rho_f + c_2 + c_6 \tilde{\lambda}_2) + (1 - \omega)(e_1 + e_6 \tilde{\lambda}_2) \} \\
& + \tilde{\gamma}_{-2} \{ \omega(g_1 \rho_f + g_2 + g_6 \tilde{\lambda}_2) + (1 - \omega)(h_1 + h_6 \tilde{\lambda}_2) \} \\
& + \tilde{\gamma}_{-1} \{ \omega(k_1 \rho_f + k_2 + k_6 \tilde{\lambda}_2) + (1 - \omega)(m_1 + m_6 \tilde{\lambda}_2) \} \left. \right] z \varepsilon_{t+1}^f \\
& + \frac{\alpha}{1 + \alpha} [\tilde{\lambda}_0 a_1 + \tilde{\lambda}_1 c_1 + \tilde{\gamma}_{-2} g_1 + \tilde{\gamma}_{-1} k_1] \omega z \varepsilon_{t+2}^f \\
& + \left[\frac{-\alpha}{1 + \alpha} \gamma \sigma_t^2 \rho_b + \frac{\alpha \tilde{\gamma}_0}{1 + \alpha} \left[\tilde{\lambda}_0 \{ \omega a_6 + (1 - \omega) d_6 \} + \tilde{\lambda}_1 \{ \omega c_6 + (1 - \omega) e_6 \} \right. \right. \\
& + \tilde{\gamma}_{-2} \{ \omega g_6 + (1 - \omega) h_6 \} + \tilde{\gamma}_{-1} \{ \omega k_6 + (1 - \omega) m_6 \} \left. \left. \right] \right] z \varepsilon_{t-1}^b \\
& - \frac{\alpha}{1 + \alpha} \gamma \sigma_t^2 z \varepsilon_t^b, \tag{1.12}
\end{aligned}$$

where

$$\begin{aligned}
z = & 1 / \left[1 - \frac{\alpha}{1 + \alpha} \left[\tilde{\lambda}_0 \{ \omega a_3 + (1 - \omega) d_3 \} + \tilde{\lambda}_1 \{ \omega c_3 + (1 - \omega) e_3 \} + \right. \right. \\
& \left. \left. \tilde{\gamma}_{-2} \{ \omega g_3 + (1 - \omega) h_3 \} + \tilde{\gamma}_{-1} \{ \omega k_3 + (1 - \omega) m_3 \} \right] \right].
\end{aligned}$$

From the method of undetermined coefficients, I have $\tilde{\lambda}_0 = \frac{1}{1 + \alpha(1 - \rho_f)}$, $\tilde{\gamma}_{-2} = -\frac{\alpha \gamma \sigma_t^2 \rho_b^2}{1 + \alpha(1 - \rho_b)}$. The other four coefficients can be solved by the following equations:

$$\begin{aligned}
\tilde{\lambda}_1 = & \frac{\alpha}{1 + \alpha} \left[\tilde{\lambda}_0 \{ \omega(a_1 \rho_f + a_2 + a_6 \tilde{\lambda}_2) + (1 - \omega)(d_1 + d_6 \tilde{\lambda}_2) \} \right. \\
& + \tilde{\lambda}_1 \{ \omega(c_1 \rho_f + c_2 + c_6 \tilde{\lambda}_2) + (1 - \omega)(e_1 + e_6 \tilde{\lambda}_2) \} \\
& + \tilde{\gamma}_{-2} \{ \omega(g_1 \rho_f + g_2 + g_6 \tilde{\lambda}_2) + (1 - \omega)(h_1 + h_6 \tilde{\lambda}_2) \} \\
& + \tilde{\gamma}_{-1} \{ \omega(k_1 \rho_f + k_2 + k_6 \tilde{\lambda}_2) + (1 - \omega)(m_1 + m_6 \tilde{\lambda}_2) \} \left. \right] z, \\
\tilde{\lambda}_2 = & \frac{\alpha}{1 + \alpha} [\tilde{\lambda}_0 a_1 + \tilde{\lambda}_1 c_1 + \tilde{\gamma}_{-2} g_1 + \tilde{\gamma}_{-1} k_1] \omega z, \\
\tilde{\gamma}_{-1} = & \left[\frac{-\alpha}{1 + \alpha} \gamma \sigma_t^2 \rho_b + \frac{\alpha \tilde{\gamma}_0}{1 + \alpha} \left[\tilde{\lambda}_0 \{ \omega a_6 + (1 - \omega) d_6 \} + \tilde{\lambda}_1 \{ \omega c_6 + (1 - \omega) e_6 \} \right. \right. \\
& + \tilde{\gamma}_{-2} \{ \omega g_6 + (1 - \omega) h_6 \} + \tilde{\gamma}_{-1} \{ \omega k_6 + (1 - \omega) m_6 \} \left. \left. \right] \right] z, \\
\tilde{\gamma}_0 = & -\frac{\alpha}{1 + \alpha} \gamma \sigma_t^2 z.
\end{aligned}$$

1.4 Simulation Results

Since the equilibrium exchange rate equation (1.12) is non-linear in parameters $\tilde{\lambda}_1, \tilde{\lambda}_2, \tilde{\gamma}_{-1}$, and $\tilde{\gamma}_0$, its solution can be solved by computer programming as a root finding problem. I assume that the values of parameters $\{\sigma_f^2, \sigma_{v_i}^2, \sigma_{v_j}^2, \sigma_b^2, \alpha, \gamma, \sigma_\varepsilon^2\}$ are $\{0.01, 0.02, 0.01, 0.01, 1, 1, 1\}$. Here I assume that variance of private signal of group i is greater than that of group j because the signal about future fundamentals at a longer horizon should be less precise. I will consider many cases depending on the AR processes of f_t and b_t .

1.4.1 The Coefficients of the Equilibrium Exchange Rate

Figures 1.1–1.4 display coefficients $\tilde{\lambda}_0, \tilde{\lambda}_1, \tilde{\lambda}_2, \tilde{\gamma}_{-2}, \tilde{\gamma}_{-1}$, and $\tilde{\gamma}_0$ as functions of the investor proportion, ω . These coefficients also vary according to the persistence of f_t and b_t . The coefficient of f_t , denoted by $\tilde{\lambda}_0$, and the coefficient of b_{t-2} , denoted by $\tilde{\gamma}_{-2}$ are constant. They depend on the assigned parameters. In all cases, the coefficient of ε_{t+1}^f , denoted by $\tilde{\lambda}_1$, and the coefficient of ε_{t+2}^f , denoted by $\tilde{\lambda}_2$, are positive. The coefficients of b_{t-2} , denoted by $\tilde{\gamma}_{-2}$, are negative, except for the case $\rho_b = 0$ and $\rho_f = 0$ for which $\tilde{\gamma}_{-2}$ is zero since there is no role of b_{t-2} in this case. The coefficient of ε_{t-1}^b , denoted by $\tilde{\gamma}_{-1}$, and the coefficient of ε_t^b , denoted by $\tilde{\gamma}_0$, are also negative. As $\omega \rightarrow 1$, $\tilde{\lambda}_1$ is decreasing but $\tilde{\lambda}_2$ is increasing in value since in the two-period model, ε_{t+2}^f has a significant role on the exchange rate. When $\omega = 0$, it is the one-period horizon model. There are no role of ε_{t+2}^f in determining the exchange rate. $\tilde{\lambda}_2$ will be equal to zero. The magnitude of each coefficient affecting the exchange rate depends on the proportion of investors who hold different information in the economy, and the autoregressive process of f_t and b_t .

1.4.2 The Magnification Factor

The term z in equation (1.12) is called the magnification factor. It magnifies the effect of unobservable shocks on the exchange rate. Specifically, the direct effect of the shock ε_t^b on s_t is captured by $-\alpha\gamma\sigma_\varepsilon^2/(1+\alpha)$. Apart from this, the effect of the hedge trade shock on the exchange rate is also amplified by the change in future macroeconomic fundamentals, through $\bar{E}_t\varepsilon_{t+1}^f, \bar{E}_t\varepsilon_{t+2}^f$,

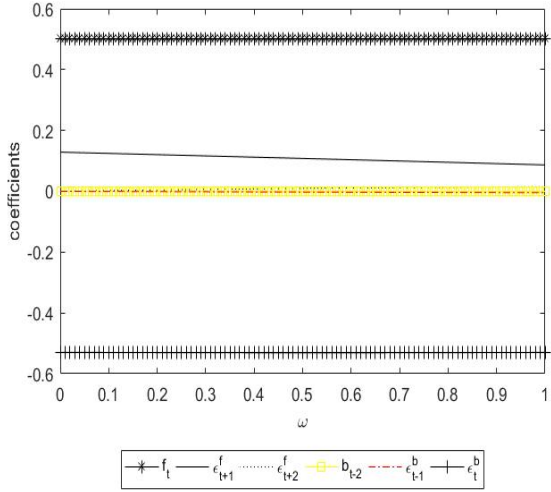


Figure 1.1: Coefficients, $\rho_b = \rho_f = 0$

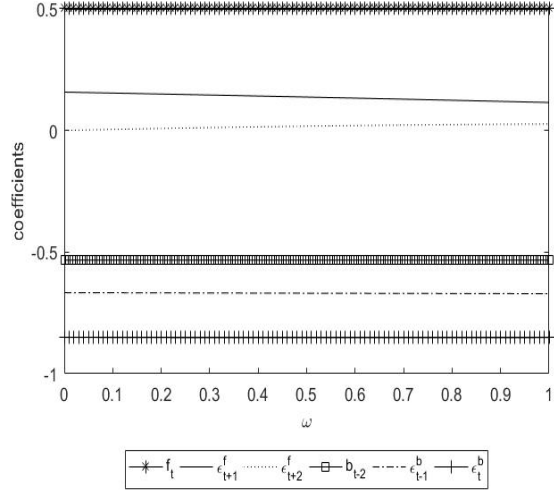


Figure 1.2: Coefficients, $\rho_b = 0.8 \rho_f = 0$

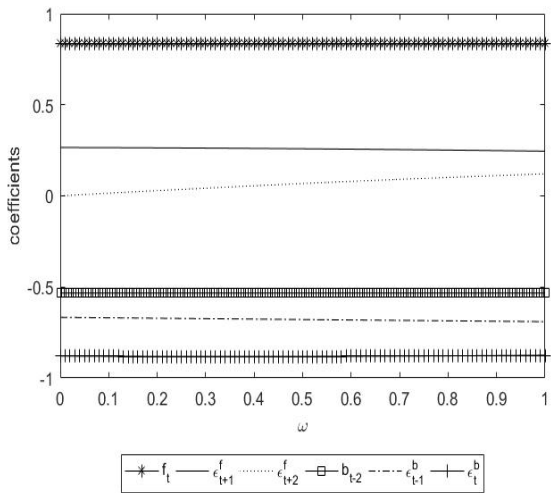


Figure 1.3: Coefficients, $\rho_b = \rho_f = 0.8$

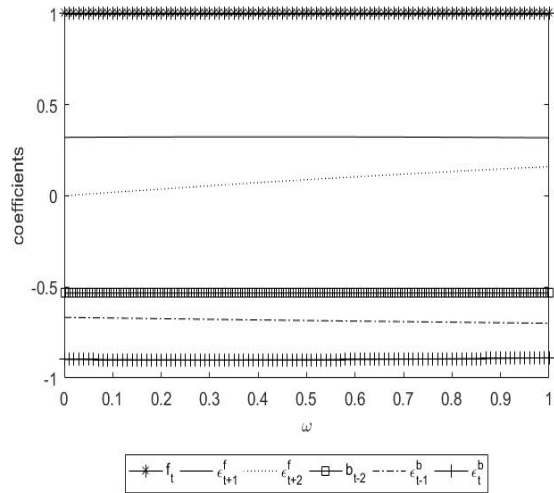


Figure 1.4: Coefficients, $\rho_b = 0.8 \rho_f = 1$

$\bar{E}_t \varepsilon_{t-1}^b$ and $\bar{E}_t \varepsilon_t^b$. Investors have rational confusion in that they do not know whether the change in exchange rate is driven by the change in private signals about future fundamentals or from the hedge trade. Suppose there is a change in the b-shock, the exchange rate is directly altered by the shock. Investors also revise their average expectation of shocks as a result of a change in exchange rate from the shock. The exchange rate is adjusted again from the impact of investors' revision in fundamentals. In this way, the exchange rate is therefore magnified. The rational confusion will last for T periods. Investors will give more weight to the exchange rate in forming average expectation of future fundamentals, until the final f_{t+T} is observed. Until that time, the effect of the b-shock continues to impact the exchange rate accordingly. In the long run, investors learn more about future fundamentals by both observing them and by receiving new private signals, the impact of the b-shock on the exchange rate recedes.

Figures 1.5–1.8 plot the values of z in response to the proportion of investors. In all cases, the values of z exceed 1. It thus magnifies the effect of unobserved shocks on exchange rate. When the process of f_t and/or b_t are more persistent, the values of z shift up. For example, when $\omega = 0$, z increases from 1.059 to 1.701 as ρ_b rises from 0 to 0.8. z also increases from 1.059 to 1.150 as ρ_f rises from 0 to 0.8. The persistence of the b-shock induces difficulty for investors to learn about future fundamentals. Then the rational confusion is more persistent so the effect on exchange rate is magnified.

The values of z increase as ω increases. However, when ω reaches a certain value, the values of z decrease. This is because the movement of z depends on the precision of the exchange rate signal. When the exchange rate signal is precise, investors give more weight to the exchange rate in forming expectations of future shocks, then z is large. However, if the private signal is imprecise, investors put a lower weight on future fundamentals in the exchange rate. This reduces the precision of the exchange rate signal, which decreases z .

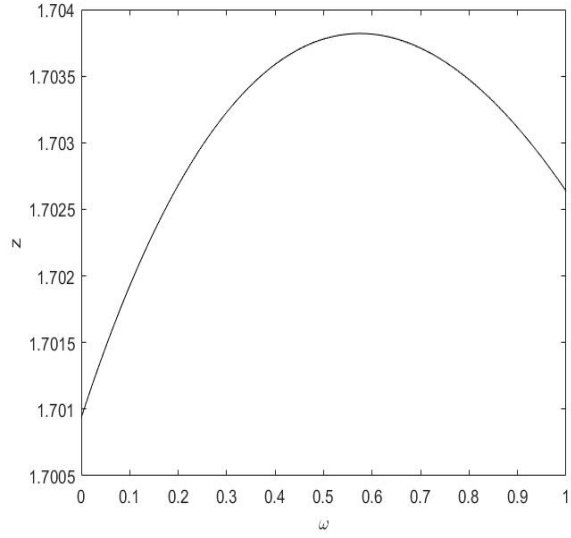
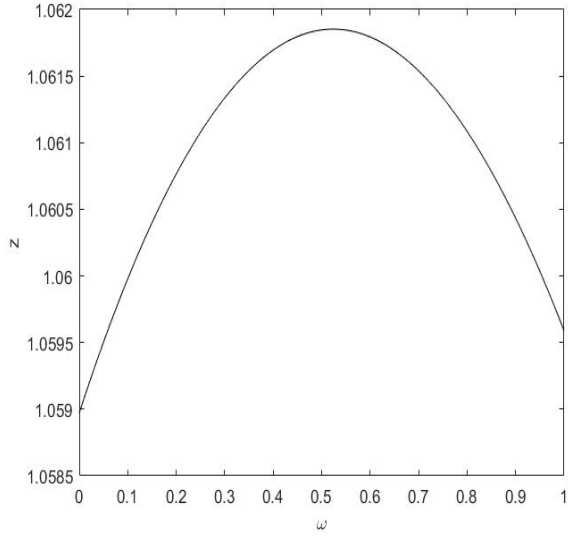


Figure 1.5: Magnification Factor, $\rho_b = \rho_f = 0$ Figure 1.6: Magnification Factor, $\rho_b = 0.8 \rho_f = 0$

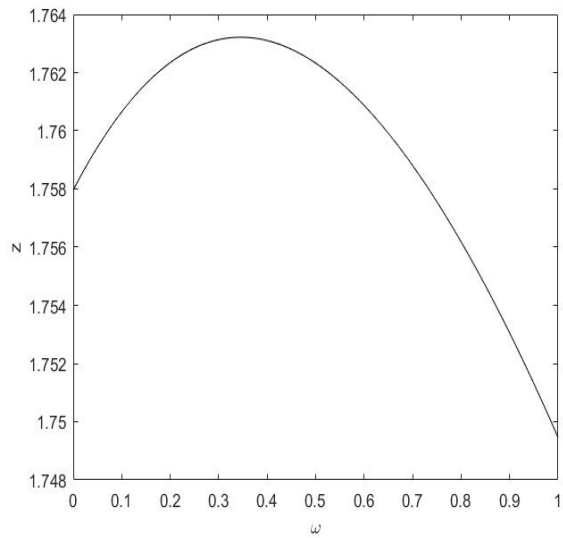
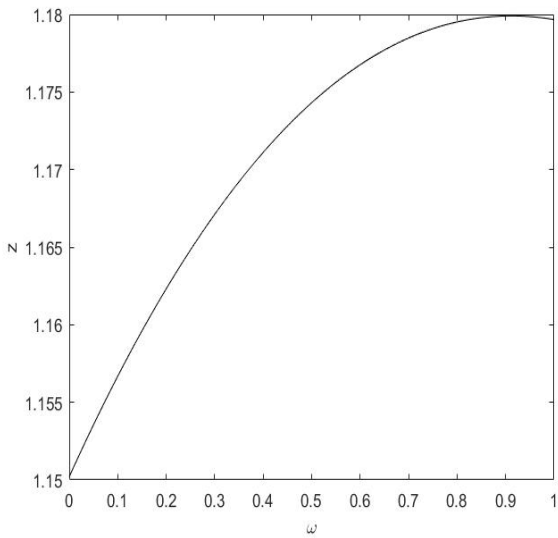


Figure 1.7: Magnification Factor, $\rho_b = 0 \rho_f = 0.8$ Figure 1.8: Magnification Factor, $\rho_b = \rho_f = 0.8$

1.4.3 The Effect of Fundamental Shocks on Exchange Rates

Figures 1.9, 1.11, and 1.13 shows the impact on the exchange rate in response to one-standard-deviation shocks in observed fundamentals. An f-shock, denoted by ε_{t+T}^f , will have first effect on the exchange rate at time t . Because we assume that money supply in home country is constant but foreign money supply is stochastic, an f-shock can be caused by an increase in the foreign interest rate. An increase in the foreign interest rate is associated with a depreciation in the home currency and a decrease in the expected future exchange rate.

When the fundamentals are more persistent, an f-shock stimulates an increase in the level of the exchange rate in the sense that the increase in exchange rate is highest in the case where all investors receive two-period ahead signals ($\omega = 1$), followed by the case where one group of investors receives two-period ahead signals and another group receives one-period ahead signals ($\omega = 0.5$), and the case where all investors receive one-period ahead signals ($\omega = 0$), respectively. Figures 1.9, 1.11, and 1.13 depict this result. An increase in the fraction of investors who know more about future fundamentals increases the fluctuation of the exchange rate in the market. The implication is that to decrease the volatility of exchange rates in the market, it should have decreased a proportion of investors who hold information about future fundamentals in advance.

The effects of a b-shock on exchange rate, shown in figures 1.10, 1.12, and 1.14, are large in the first period, but its impact dies down with an increasing of time period. The effect of f- and b-shocks on the exchange rate depend deeply on their stochastic processes. For example, if $\rho_b = \rho_f = 0.8$, the effect of shocks on the exchange rate gradually diminishes according to time. But when $\rho_b = \rho_f = 0$, the exchange rate reaches zero after it hits the highest level. Lastly, if f_t has a unit root, the exchange rate keeps constant for a certain time.

1.5 Exchange Rate Disconnect Puzzle in Heterogeneous Informational Horizons Framework

The exchange rate disconnect puzzle can be explained by information heterogeneity. Bacchetta and van Wincoop (2006) show that the R^2 of the regression of the exchange rate on observed

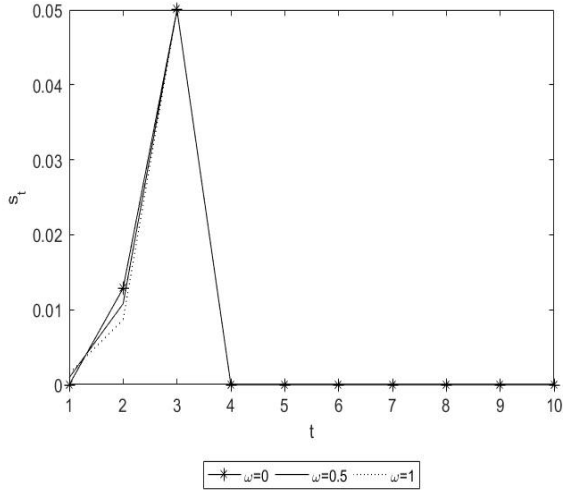


Figure 1.9: F-Shock, $\rho_b = \rho_f = 0$

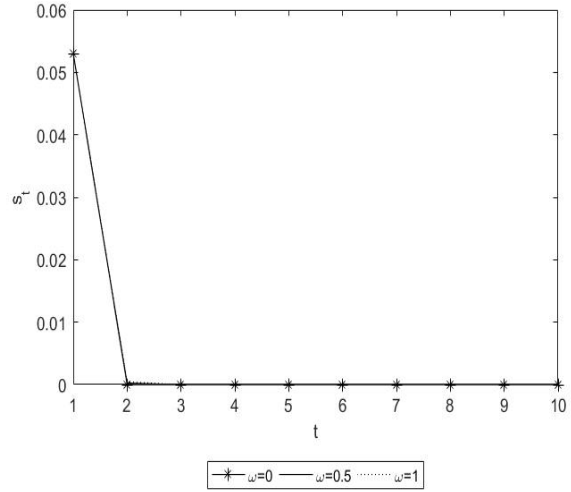


Figure 1.10: B-Shock, $\rho_b = \rho_f = 0$

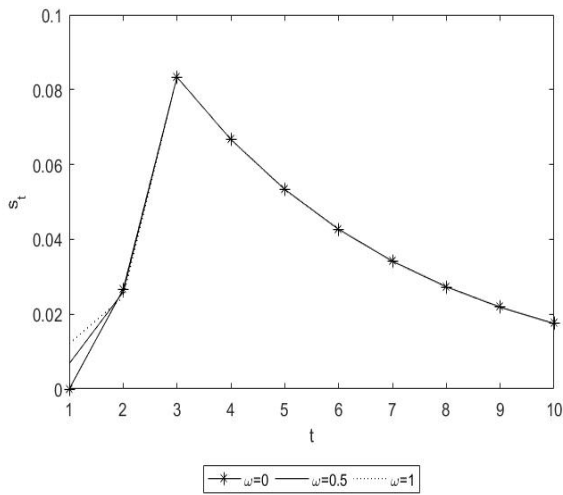


Figure 1.11: F-Shock, $\rho_b = \rho_f = 0.8$

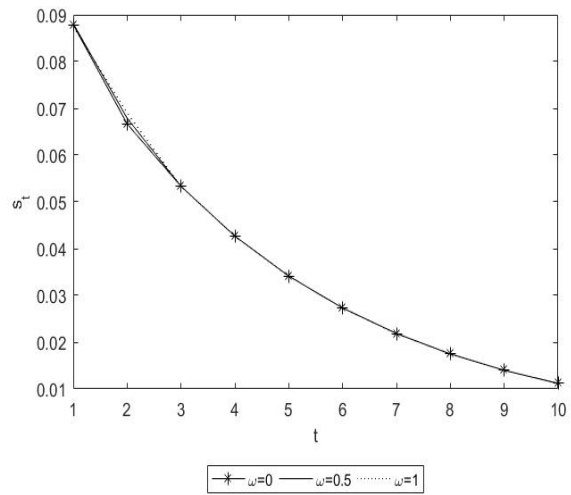
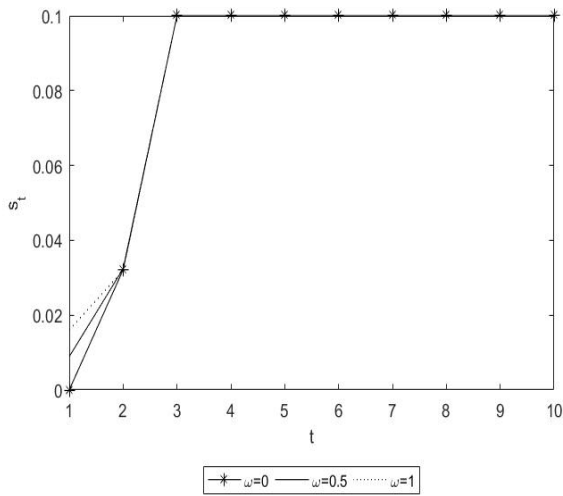
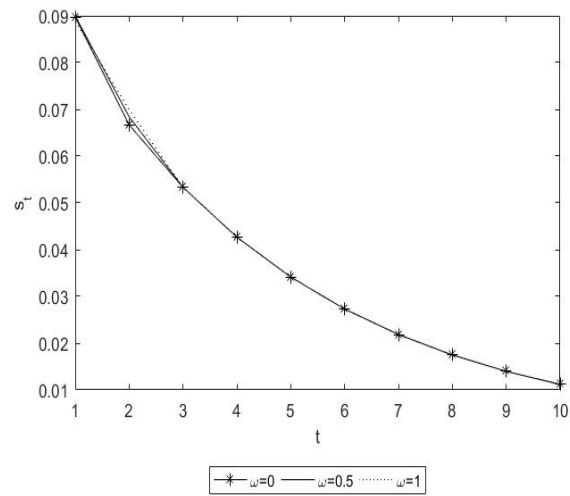


Figure 1.12: B-Shock, $\rho_b = \rho_f = 0.8$

Figure 1.13: F-Shock, $\rho_b = 0.8$ $\rho_f = 1$ Figure 1.14: B-Shock, $\rho_b = 0.8$ $\rho_f = 1$

fundamentals in the heterogeneous model is lower than in the common knowledge model. Investors in the economy have different beliefs about future macro fundamentals. These fundamentals are not directly observed, investors infer about them from the private signals they receive. The private information of investors is aggregated in the exchange rate. With unobserved shocks, they will be preventing the average private signal from being fully revealed by the exchange rate.

There are three scenarios to study the disconnect between the exchange rate and observed fundamentals in a heterogeneous informational setting. The first one is the case when $\omega = 0$, where all investors receive one-period ahead signal about future fundamentals. The second one is when $\omega = 0.5$. This is a heterogeneous informational horizon case in which one group of investors receives two-period ahead signals and another group receives one-period ahead signals about future fundamentals. And the last one is $\omega = 1$. This is a situation where all investors only receive two-period ahead signals.

To examine the disconnect between the exchange rate and the observed fundamentals of these scenarios, I will compare the R^2 of a regression of the exchange rate on observed fundamentals in each case. The R^2 is computed as follows:

$$R^2 = \frac{\tilde{\lambda}_0^2 \sigma_f^2 / (1 - \rho_f^2)}{(\tilde{\lambda}_1^2 + \tilde{\lambda}_2^2) \sigma_f^2 + (\tilde{\gamma}_0^2 + \tilde{\gamma}_{-1}^2) \sigma_b^2 + \tilde{\lambda}_0^2 \sigma_f^2 / (1 - \rho_f^2) + \tilde{\gamma}_{-2}^2 \sigma_b^2 / (1 - \rho_b^2)}.$$

A high value of the R^2 indicates that variance of the exchange rate is mostly captured by the variance of the current observable fundamental, implying that there is a tight relationship between the exchange rate and the current observed fundamental (f_t). When the R^2 is high, the disconnect puzzle attenuates.

Obviously, the value of R^2 depends on the precision of the private signals, whose effects are through the coefficients of the equilibrium exchange rate, the variances of f_t and b_t errors, and the persistence of f_t and b_t . I will focus on the effect of signal precisions and the effect of persistence on R^2 , while I keep the variances of f_t and b_t errors equal ($\sigma_f^2 = \sigma_b^2 = 0.01$).

Figures 1.15–1.18 plot movements of R^2 in response to the degree of signal precision in various degree of fundamentals persistence. The values of R^2 in these figures differ according to the degree of the fundamentals persistence. In each figure, there are three lines. The dotted line plots the R^2 where $\sigma_{v_i}^2 = 2\sigma_{v_j}^2$. The solid line plots the R^2 where $\sigma_{v_i}^2 = \sigma_{v_j}^2$. The starred line plots R^2 where $\sigma_{v_i}^2 = 0.5\sigma_{v_j}^2$. As you can see, the R^2 decreases as $\omega \rightarrow 1$, except the dotted lines in figures 1.15–1.16. That is, when a group of investors holds different beliefs about future fundamentals, i.e. scenario 2, the disconnect problem is stronger than in scenario 1 in which investors hold information about future fundamentals at the shortest horizon. Moreover, the disconnect problem is strongest in scenario 3, where investors perceive information about future fundamentals at the longest period. That is, the disconnect puzzle is more serious when there is a difference in informational horizon received among investors, and that problem is even more severe when investors receive information about future fundamentals at a long horizon.

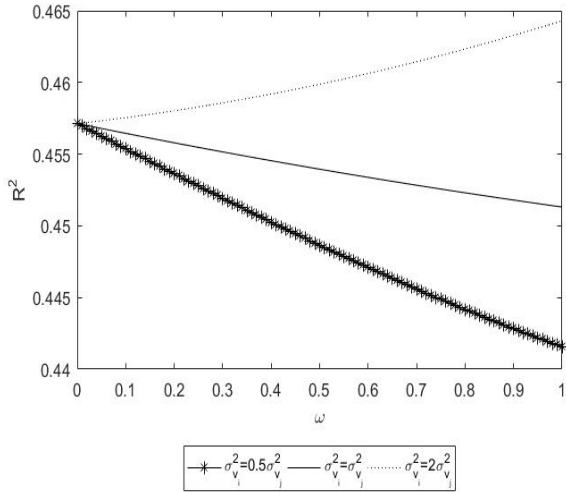


Figure 1.15: R^2 , $\rho_b = \rho_f = 0$

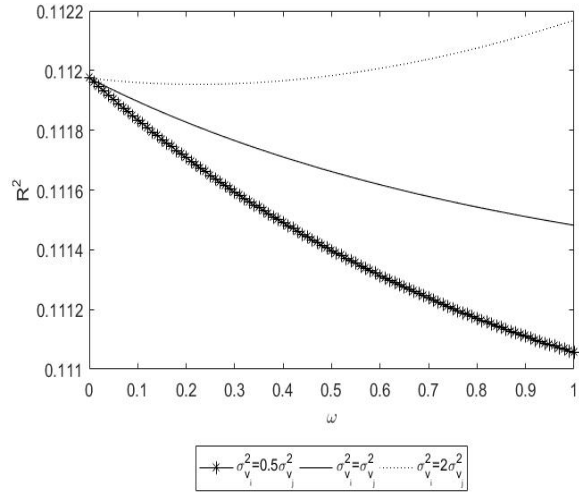


Figure 1.16: R^2 , $\rho_b = 0.8$ $\rho_f = 0$

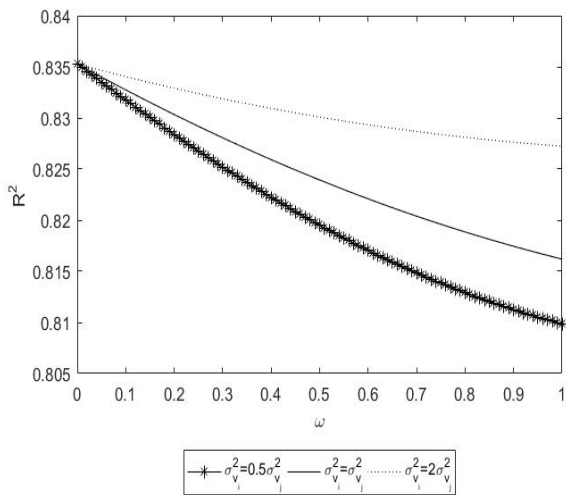


Figure 1.17: R^2 , $\rho_b = 0$ $\rho_f = 0.8$

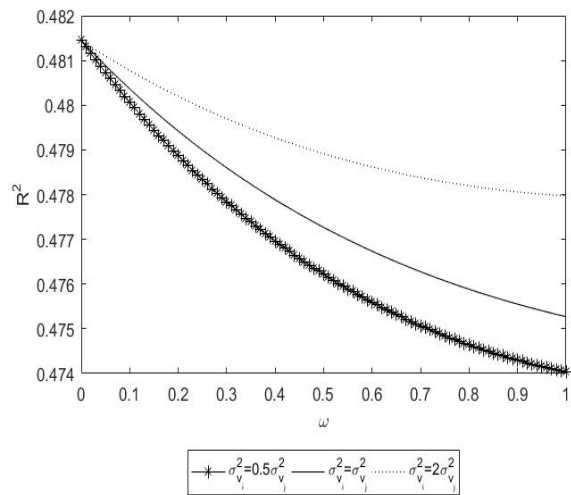


Figure 1.18: R^2 , $\rho_b = \rho_f = 0.8$

1.5.1 The Effect of Degree of Persistence

An increase in the persistence of f_t shifts the level R^2 up. This is illustrated by comparing the R^2 in figure 1.15 with figure 1.17 or figure 1.16 with figure 1.18. This is because when f_t is persistent, the past of f_t also helps forecasting and giving information about future f_t . The exchange rate will be more connected to the current fundamental when future fundamentals are more likely known.

However, the level of R^2 drops when ρ_b increases. The evidence is shown in figures 1.15–1.16 and figures 1.17–1.18, where ρ_b increases from 0 to 0.8. When b_t is more persistent, the impact of the unobservable components on the exchange rate lasts longer. Also, the rational confusion will be more persistent. This causes the disconnect between exchange rate and observed fundamental.

Bacchetta and van Wincoop (2006) find that the R^2 in the heterogeneous information model is relatively lower than in the common knowledge model. They conclude that the presence of private information caused the disconnect between exchange rate and current observed fundamental. My results are consistent with Bacchetta and van Wincoop's finding that heterogeneous information causes exchange rate to disconnect with current observed fundamentals. Moreover, I find that the disconnect is intense when one group of investors is early informed. The problem is more serious when a fraction of investors receiving information about these future fundamentals at a longer period increases. An increase in a fraction of early-informed investors worsens the disconnect problem. When there are more investors who are early informed, the unobservable components in the model, i.e. future fundamentals f_{t+j} , and unobserved fundamentals b_{t-j} , become more significant. Since the exchange rate equilibrium is composed of these unobservable parts, therefore, the exchange rates are more disconnected from the current observable fundamental f_t . The larger the fraction of early-informed investors, the greater the influence of the unobserved components on the exchange rate so that it dissociates with the current observable fundamental. However, the exchange rate will have more connection with the current fundamental if the fundamental is persistent. When the process of fundamental f_t is persistent, the future fundamental f_{t+j} highly

correlates with the current f_t . Knowing more about future fundamental would help connect the exchange rate with the current observable fundamental.

1.5.2 The Effect of Private Signal Precision

As $\omega \rightarrow 1$, the unobservable components in the R^2 are more significant. For example, f_{t+2} will be more significant if there is an increase in the proportion of investors knowing information about future fundamentals two periods ahead. This does cause a rise in $\tilde{\lambda}_2$, which finally brings down the R^2 . However, an increase in the proportion of investors who know more about future fundamentals induces an increase in the proportion of investors who have private signals of bad quality. This could interfere with the movement of R^2 .

The rationale of movements in R^2 depends deeply on the structure of private information. Let us consider 3 cases which are a) $\sigma_{v_i}^2 = 2\sigma_{v_j}^2$, b) $\sigma_{v_i}^2 = \sigma_{v_j}^2$, and c) $\sigma_{v_i}^2 = 0.5\sigma_{v_j}^2$. In case (a), it is ambiguous for group i investors whether they have an informational advantage over group j as they receive information about future fundamentals earlier but with higher signal error. In cases (b) and (c), group i investors are better informed in the sense that they obtain information about future fundamentals in advance of the remaining investors with equal or lower signal error.

The exchange rate disconnect puzzle could be complicated by the precision of the private signal received by investors. The conclusion that the disconnect problem worsens as $\omega \rightarrow 1$ could not be displayed in figures 1.15–1.16 under the case where group i investors have higher variance of signal error (graphed by a dotted line). Even though they are early informed about future fundamentals by receiving information about future fundamentals at a longer period than other investors, their signals are less precise. That is, these investors are early informed about future fundamentals, but the signal they receive has more noise. In this case, the R^2 will not decrease as $\omega \rightarrow 1$. The conclusion about the disconnect problem is vague when it is difficult to say about who has informational advantage. In section 1.6, I will develop a new structure of private signal and discuss the change of results.

1.6 A Revisit of Information Structure

In this section, the information structure is changed in order to provide an informational advantage between group of investors.

1.6.1 Revised Structure

In the previous structure, group i investors receive a private signal about two-period ahead of fundamental f . This means at time t they receive information about f_{t+2} . They also perceive information about f_{t+1} since time $t - 1$ while group j just knows about f_{t+1} at time t . Being an early-informed investor comes with a cost of getting higher signal error. A signal about f_{t+1} that group i investors receive would have more noise than in a signal about f_{t+1} group j could get at period t . The farther is period of information about future fundamentals, the higher is the variance of the signal error. The imprecision of the signal reduces the advantage of being an early-informed investor. These investors will not be better-informed than the others. I improve the information structure received by group i investors such that at time t , not only they receive a signal about future fundamental f_{t+2} , but they also receive a signal about f_{t+1} with the same signal variance as group j investors receive signal about f_{t+1} at time t . Investors in group i are better-informed investors. The information structure is shown as follows:

$$\begin{aligned} v_t^i &= f_{t+2} + \varepsilon_t^{v_i}, \varepsilon_t^{v_i} \sim N(0, \sigma_{v_i}^2), \\ w_t^i &= f_{t+1} + \varepsilon_t^{w_i}, \varepsilon_t^{w_i} \sim N(0, \sigma_{w_i}^2). \end{aligned}$$

Investors in group j still receive the private signal

$$v_t^j = f_{t+1} + \varepsilon_t^{v_j}, \varepsilon_t^{v_j} \sim N(0, \sigma_{v_j}^2),$$

where $\sigma_{v_i}^2 > \sigma_{w_i}^2$, $\sigma_{v_j}^2 = \sigma_{w_i}^2$.

Group i investors receive one signal about future fundamental f_{t+2} . This signal is denoted by v_t^i . They also receive two signals about future fundamental f_{t+1} . Those two signals are v_{t-1}^i and w_t^i . The aggregated signal about f_{t+1} is called y_t^i . This new signal y_t^i is a weighted average of the

two signal about f_{t+1} which are v_{t-1}^i and w_t^i , where the weight is its precision of the signal.² y_t^i can be written by

$$y_t^i = f_{t+1} + \varepsilon_t^{y_i} = \frac{\beta_{v_i} v_{t-1}^i + \beta_{w_i} w_t^i}{\beta_{v_i} + \beta_{w_i}},$$

where $\beta_{v_i} = 1/\sigma_{v_i}^2$ and $\beta_{w_i} = 1/\sigma_{w_i}^2$. Since the signal y_t^i is weighted by the precision of the signal, the signal having higher precision will be given more weight.

Let $\sigma_{y_i}^2$ be the variance of signal y_i 's error, $var(\varepsilon_t^{y_i})$, then $\beta_{y_i} = 1/\sigma_{y_i}^2 = \beta_{v_i} + \beta_{w_i}$. I can find that $\sigma_{y_i}^2 < \sigma_{v_j}^2$. That is, the variance of signal about f_{t+1} error for group i investors is less than group j investors.

The method of solving the model is similar to the previous one. I therefore have the equilibrium of exchange rate as

$$\begin{aligned} s_t = & \frac{1}{1 + \alpha(1 - \rho_f)} f_t - \frac{\alpha \gamma \sigma_t^2 \rho_b^2}{1 + \alpha(1 - \rho_b)} b_{t-2} \\ & + \frac{\alpha}{1 + \alpha} \left[\tilde{\lambda}_0 \{ \omega (a'_1 \rho_f + a'_2 + a'_7 + a_6 \tilde{\lambda}_2) + (1 - \omega)(d_1 + d_6 \tilde{\lambda}_2) \} \right. \\ & + \tilde{\lambda}_1 \{ \omega (c'_1 \rho_f + c'_2 + c'_7 + c_6 \tilde{\lambda}_2) + (1 - \omega)(e_1 + e_6 \tilde{\lambda}_2) \} \\ & + \tilde{\gamma}_{-2} \{ \omega (g'_1 \rho_f + g'_2 + g'_7 + g'_6 \tilde{\lambda}_2) + (1 - \omega)(h_1 + h_6 \tilde{\lambda}_2) \} \\ & \left. + \tilde{\gamma}_{-1} \{ \omega (k'_1 \rho_f + k'_2 + k'_7 + k_6 \tilde{\lambda}_2) + (1 - \omega)(m_1 + m_6 \tilde{\lambda}_2) \} \right] z' \varepsilon_{t+1}^f \\ & + \frac{\alpha}{1 + \alpha} [\tilde{\lambda}_0 a'_1 + \tilde{\lambda}_1 c'_1 + \tilde{\gamma}_{-2} g'_1 + \tilde{\gamma}_{-1} k'_1] \omega z' \varepsilon_{t+2}^f \\ & + \left[\frac{-\alpha}{1 + \alpha} \gamma \sigma_t^2 \rho_b + \frac{\alpha \tilde{\gamma}_0}{1 + \alpha} \left[\tilde{\lambda}_0 \{ \omega a'_6 + (1 - \omega) d_6 \} + \tilde{\lambda}_1 \{ \omega c'_6 + (1 - \omega) e_6 \} + \tilde{\gamma}_{-2} \{ \omega g'_6 + (1 - \omega) h_6 \} \right. \right. \\ & \left. \left. + \tilde{\gamma}_{-1} \{ \omega k'_6 + (1 - \omega) m_6 \} \right] \right] z' \varepsilon_{t-1}^b \\ & - \frac{\alpha}{1 + \alpha} \gamma \sigma_t^2 z' \varepsilon_t^b, \end{aligned} \tag{1.13}$$

where

$$\begin{aligned} z' = & 1/ \left[1 - \frac{\alpha}{1 + \alpha} [\tilde{\lambda}_0 \{ \omega a'_3 + (1 - \omega) d_3 \} + \tilde{\lambda}_1 \{ \omega c'_3 + (1 - \omega) e_3 \} \right. \\ & \left. + \tilde{\gamma}_{-2} \{ \omega g'_3 + (1 - \omega) h_3 \} + \tilde{\gamma}_{-1} \{ \omega k'_3 + (1 - \omega) m_3 \} \right]. \end{aligned}$$

²The aggregate signal about f_{t+1} , denoted by y_t^i , is a linear aggregation between v_{t-1}^i and w_t^i as $y_t^i = r v_{t-1}^i + (1 - r) w_t^i$, where r is the weight chosen to minimize $var_t(y_t^i)$. The optimal weight $r^* = \frac{\beta_{v_i}}{\beta_{v_i} + \beta_{w_i}}$, where $\beta_{v_i} = 1/\sigma_{v_i}^2$ and $\beta_{w_i} = 1/\sigma_{w_i}^2$.

From the method of undetermined coefficients, I have $\tilde{\lambda}_0 = \frac{1}{1+\alpha(1-\rho_f)}$, $\tilde{\gamma}_{-2} = -\frac{\alpha\gamma\sigma_t^2\rho_b^2}{1+\alpha(1-\rho_b)}$. The other four coefficients can be solved by the following equation:

$$\begin{aligned}\tilde{\lambda}_1 &= \frac{\alpha}{1+\alpha} \left[\tilde{\lambda}_0 \{ \omega(a'_1\rho_f + a'_2 + a'_7 + a'_6\tilde{\lambda}_2) + (1-\omega)(d_1 + d_6\tilde{\lambda}_2) \} \right. \\ &\quad + \tilde{\lambda}_1 \{ \omega(c'_1\rho_f + c'_2 + c'_6\tilde{\lambda}_2) + (1-\omega)(e_1 + e_6\tilde{\lambda}_2) \} \\ &\quad + \tilde{\gamma}_{-2} \{ \omega(g'_1\rho_f + g'_2 + g'_7 + g'_6\tilde{\lambda}_2) + (1-\omega)(h_1 + h_6\tilde{\lambda}_2) \} \\ &\quad \left. + \tilde{\gamma}_{-1} \{ \omega(k'_1\rho_f + k'_2 + k'_7 + k'_6\tilde{\lambda}_2) + (1-\omega)(m_1 + m_6\tilde{\lambda}_2) \} \right] z', \\ \tilde{\lambda}_2 &= \frac{\alpha}{1+\alpha} [\tilde{\lambda}_0 a'_1 + \tilde{\lambda}_1 c'_1 + \tilde{\gamma}_{-2} g'_1 + \tilde{\gamma}_{-1} k'_1] \omega z', \\ \tilde{\gamma}_{-1} &= \left[\frac{-\alpha}{1+\alpha} \gamma \sigma_t^2 \rho_b + \frac{\alpha \tilde{\gamma}_0}{1+\alpha} \left[\tilde{\lambda}_0 \{ \omega a'_6 + (1-\omega) d_6 \} + \tilde{\lambda}_1 \{ \omega c'_6 + (1-\omega) e_6 \} + \tilde{\gamma}_{-2} \{ \omega g'_6 + (1-\omega) h_6 \} \right. \right. \\ &\quad \left. \left. + \tilde{\gamma}_{-1} \{ \omega k'_6 + (1-\omega) m_6 \} \right] \right] z', \\ \tilde{\gamma}_0 &= -\frac{\alpha}{1+\alpha} \gamma \sigma_t^2 z' .\end{aligned}$$

In addition to the variance of one-period signal error of group i , the value of parameters are assumed to be the same as $\{\sigma_f^2, \sigma_{v_i}^2, \sigma_{v_j}^2, \sigma_b^2, \alpha, \gamma, \sigma_t^2, \sigma_{w_i}^2\} = \{0.01, 0.02, 0.01, 0.01, 1, 1, 1, 0.01\}$. Here I assume that the variance of one-period ahead signal of group i and group j are equal, ($\sigma_{w_i}^2 = \sigma_{v_j}^2 = 0.01$). The variance of two-period private signal of group i is greater and is assigned to $\sigma_{v_i}^2 = 0.02$. Coefficients $\tilde{\lambda}_0$, $\tilde{\lambda}_1$, $\tilde{\lambda}_2$, $\tilde{\gamma}_{-2}$, $\tilde{\gamma}_{-1}$, and $\tilde{\gamma}_0$ are graphed in Figures 1.19–1.22. The direction of coefficients does not diverge from the the previous setting. As $\omega \rightarrow 1$, $\tilde{\lambda}_1$ and $\tilde{\lambda}_2$ are increasing. Similarly, the magnitude of each coefficient affecting the exchange rate depends on the proportion of investors who hold different information in the economy, and the autoregressive process of f_t and b_t .

Figures 1.23–1.26 plot the value of z in response to the proportion of investors. Since z is a multiplier, its value exceeds 1. The z term multiplies the effect of unobserved shocks on exchange rate. The direction of z depends on the persistence of f_t and b_t . When b_t is more persistent, z is decreasing. When f_t is more persistent, z is initially increasing up to a certain point then it decreases.

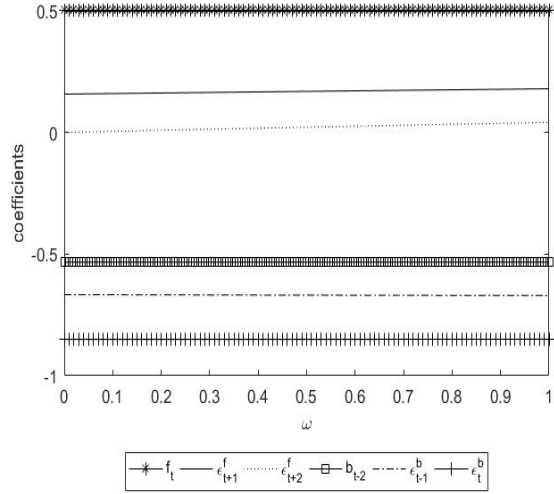
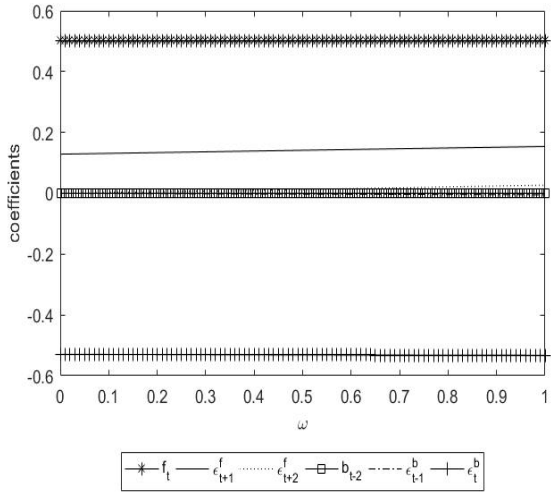


Figure 1.19: Revised Coefficients, $\rho_b = \rho_f = 0$ Figure 1.20: Revised Coefficients, $\rho_b = 0.8$, $\rho_f = 0$

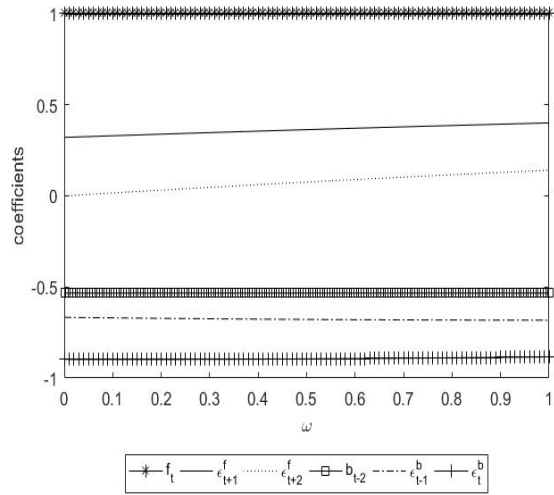
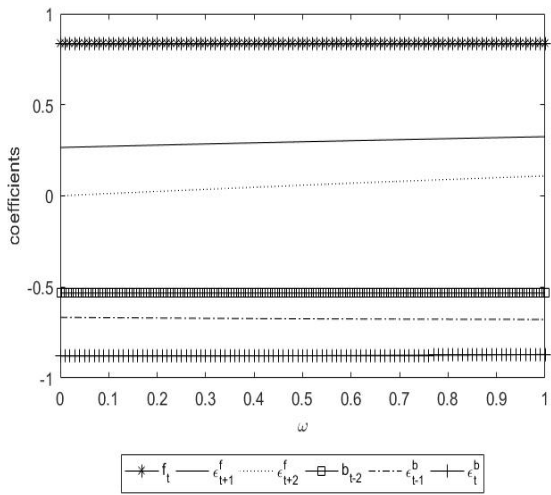


Figure 1.21: Revised Coefficients, $\rho_b = \rho_f = 0.8$ Figure 1.22: Revised Coefficients, $\rho_b = 0.8$, $\rho_f = 1$

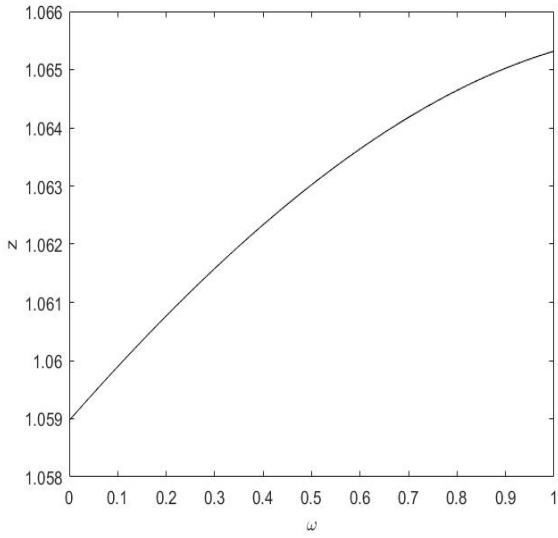


Figure 1.23: Revised Magnification Factor, $\rho_b = \rho_f = 0$

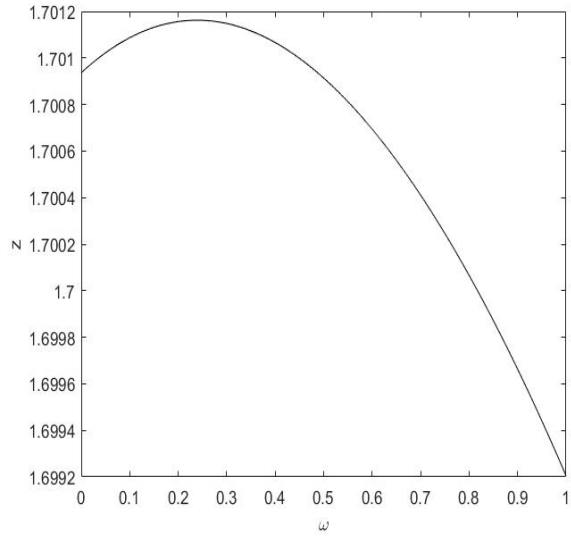


Figure 1.24: Revised Magnification Factor, $\rho_b = 0.8, \rho_f = 0$

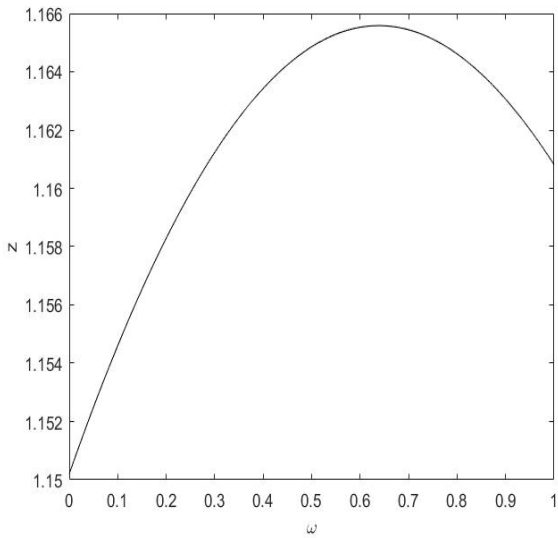


Figure 1.25: Revised Magnification Factor, $\rho_b = 0, \rho_f = 0.8$

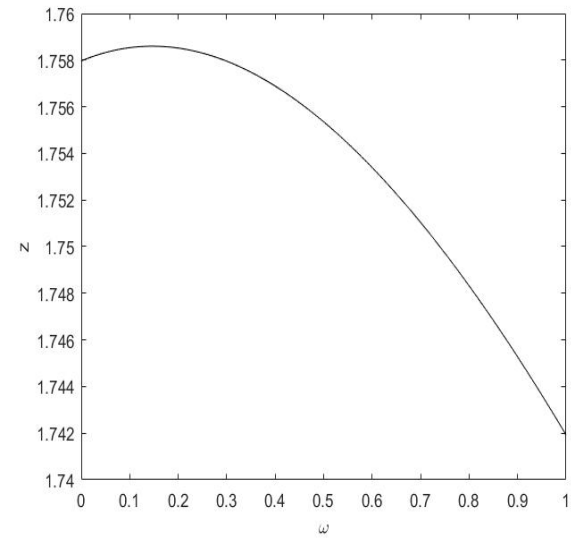


Figure 1.26: Revised Magnification Factor, $\rho_b = 0.8, \rho_f = 0.8$

1.6.2 The Role of Heterogeneous Informational Horizons and the Exchange Rate Disconnect with Current Observable Fundamental

With the information structure proposed in this section, an increase in the proportion of the better-informed investors would make the exchange rate disconnect puzzle more serious, regardless of the degree of fundamentals persistence. Figures 1.27–1.30 plot the R^2 between the exchange rate and the current observable fundamental. Each figure is differentiated by the degree of the fundamentals' persistence. All of these figures illustrate that the R^2 diminishes as $\omega \rightarrow 1$. That is, as investors perceive more information at a longer period ahead, the more severe is the disconnect problem. As I mentioned earlier, when $\omega \rightarrow 1$, the unobserved components in the model, i.e. future fundamentals, become more significance, then the R^2 of the exchange rate and the current observable fundamental is low. However, when f_t is more persistent, the values of R^2 shift up. As future fundamental highly correlates with current fundamental, the exchange rate would be more connected with the current fundamental.

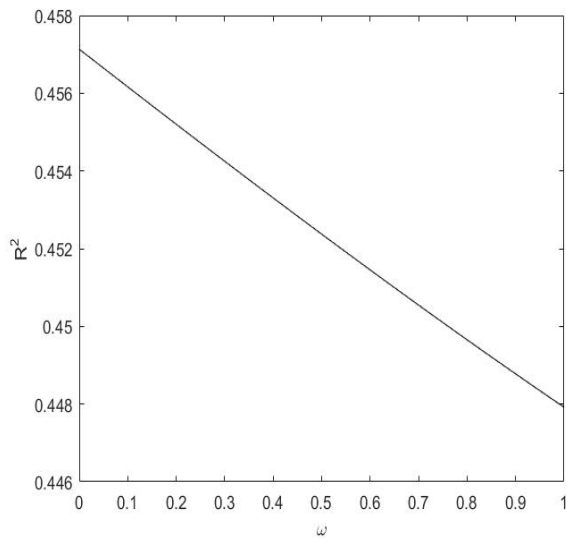


Figure 1.27: Revised R^2 , $\rho_b = \rho_f = 0$

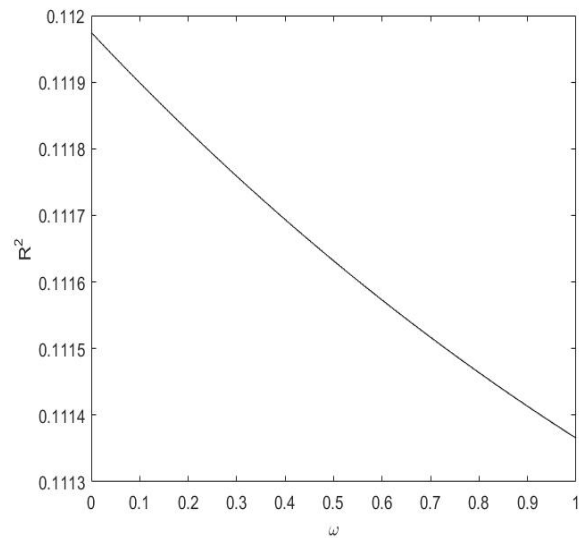
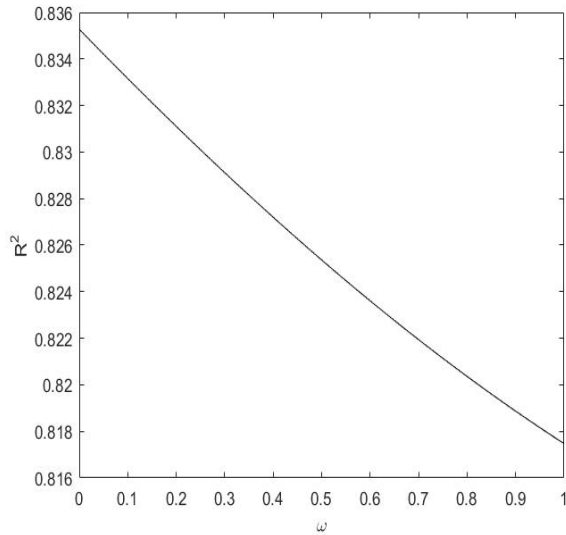
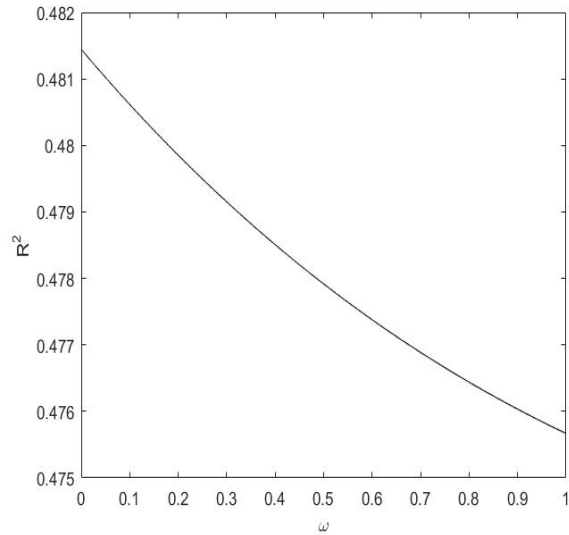


Figure 1.28: Revised R^2 , $\rho_b = 0.8$, $\rho_f = 0$

Figure 1.29: Revised R^2 , $\rho_b = 0$ $\rho_f = 0.8$ Figure 1.30: Revised R^2 , $\rho_b = \rho_f = 0.8$

The disconnect problem is more intense when investors receive signal with heterogeneous informational horizon. The dotted line in figure 1.31 plots R^2 of the model where all investors symmetrically receive one-period ahead signal, and the variance of signal error equals. The R^2 is flat at 0.4814. The starred line displays R^2 of the model where there are two group of investors receiving one-period signal with unequal variance of signal error. As $\omega \rightarrow 1$, the proportion of investors with lower variance of signal error increases. As seen in the figure, R^2 keeps decreasing as $\omega \rightarrow 1$. An increase in the proportion of investors with more precise signal makes the disconnect worsen. When signals are more precise, investors learn more about future fundamentals clearer. Therefore the current observable fundamental has little explanatory power about the exchange rate. The solid line represents the R^2 where one group of investors receives only one-period signal and the other group receives both one-, and two-period signals. With different signal horizons, the value of R^2 is the lowest among the three cases. Moreover, the disconnect problem is stronger as the proportion of investors receiving two-period ahead signal increases. In fact, the disconnect problem is not only about the different in precision of the private signal. The introduction of the heterogeneous informational horizon in the model would make the disconnect problem more severe.

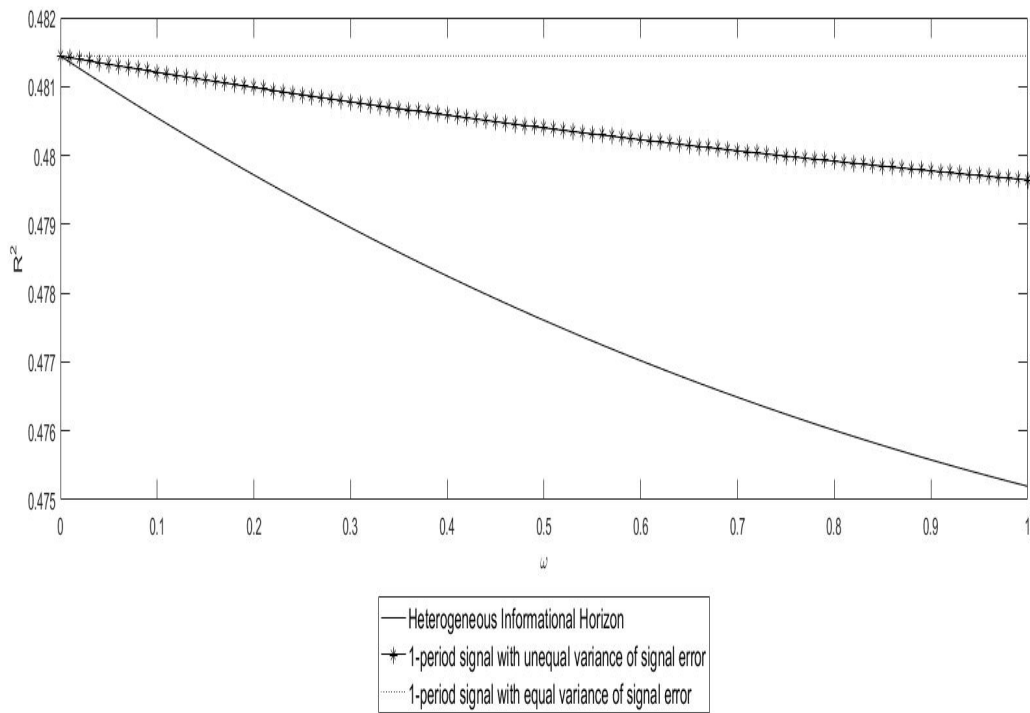


Figure 1.31: R^2 in Heterogeneous Information Framework

1.7 Exchange Rate and Order Flow

1.7.1 The Structure of the Foreign Exchange Market

The foreign exchange market is two-tiered. In the first tier, called “retail market”, dealers trade with customers. The second tier concerns interdealer or interbank trades. The interdealer market plays a vital role in the foreign exchange market since retail prices are based on information from interdealer prices. In general, trading occurs electronically between participants. The market has no need to be at a specific physical location and therefore trade occurs around the clock. The posted bid-offer quotes between dealers become the benchmark for market prices. The foreign exchange market is quote-driven in which instantaneous liquidity is provided by market makers. Currency dealers have no obligation to provide liquidity in the market. However, in practice, failure to provide liquidity is costly to them as they could lose reputation.

The customers in the market can be categorized into two groups, namely, financial customers and corporate customers. All of them are institutions, unlike trading in stock markets in which individuals can trade on their own. Financial customers are mostly asset managers including leveraged investors such as hedge funds and commodity traders, as well as mutual funds and pension funds. Financial customers also include non-dealing banks, central banks, and other financial institutions. Corporate customers refer to commercial firms who demand foreign currency for their production, investment, and international trade. Financial customers are major customers in the market. Over 60% of foreign currency trading is from financial customers. Besides, they tend to have larger transactions on average than corporate customers.

Interdealer trading effectively governs the spot foreign exchange market. The interdealer trading is electronically executed by Electronic Broking Service (EBS) and Thomson Reuters where market prices of foreign currency are based on posted bid and ask quotes. Dealers indirectly trade with each other by placing market and limit orders to buy or sell currencies via electronic brokerages. The system will prioritize the limit orders to match the best prices with the incoming market orders. Interdealer quotes are constrained by position and loss limits due to dealers’ preferences

for holding zero inventories overnight. When dealers trade with retail customers, the quote price will be set from interdealer quotes and adjust from there. However, the customer market is not transparent because quotes and transactions depend on private information between the customer and the dealer. Agents supply liquidity by placing a limit order, while agents who demand liquidity enter with a market order. At the end of a trading day, dealers will manage their inventory to be close to zero. This is done mostly via interdealer trades. Through these trades, dealers quickly eliminate their inventory position. For example, the half-life of an inventory in the NYSE is over a week (Madhavan and Smidt, 1993) while the half-life of an inventory position is below five minutes for highly active dealers and below half an hour for less active dealers (Bjønnes and Rime, 2005). Dealers' inventory management affects the volatility of exchange rates as they move to accommodate inventory adjustment.

1.7.2 Understanding Order Flow

In the market microstructure approach, it is common to believe that private information is transmitted to the market via order flow. Order flow, the net of buyer-initiated orders and seller-initiated orders, generated by foreign exchange trade between investors and dealers can convey information to dealers about the state of the macroeconomy. Dealers use this information to revise their quotes price on exchange rates. The dynamics of the spot exchange rate is rather driven by the flow of information about the economy that reaches dealers than the effect of macroeconomic shocks. The role of order flow has been first highlighted by Evans and Lyons (2002). They find that order flow helps determine the price of the foreign exchange because price information is aggregated in the order flow through the trading of foreign exchange process. They also find that order flow accounts for more than 60% of daily changes in the log deutsche mark/dollar movements, and 40% of the change in log yen/dollar movements.

The portfolio shifts model (Evans and Lyons, 2002) is constructed to explain how order flow drives price determination via information aggregation. In the model, there are three rounds of trading within each day. In round 1, dealers trade with their customers. All market participants

observe the flow of publicly available macroeconomic information. Each dealer quotes a price at which he agrees to buy/sell any amount. He receives customer orders at his quoted price. Only a recipient dealer can observe his customer orders. Since customer orders are not publicly observed, they become a vital source of private information.

In round 2, each dealer trades with other dealers at his quoted price. The quotes are observable and available to all dealers. At the end of this round, all agents observe the interdealer order flows. Interdealer order flow is positive (negative) when a dealer initiating purchases (sells) foreign exchange at the ask (bid) quote. In round 3, dealers trade with public customers again. The quotes in this round are observable and available to the public. However, in the actual trading process, dealers quote prices, receive orders, and execute their trades on a continual basis.

The price of trading is conditioned on common information. At the beginning of round 1, the payoff increment, i.e., the change in the nominal interest differential, is common information. Order flows are not observed until the end of round 2. The price of round 3 reflects information in both order flows and the payoff increment. Each interdealer trading in round 2 will be proportional to his customer orders, which are only observed by the dealer who trades with its customers in round 1, as the dealer tries to manage his inventory position close to zero. Hence, when dealers observe (interdealer) order flows at the end of round 2, they can infer the aggregate portfolio shifts or customer orders in round 1. This is how information about aggregate demand for foreign exchange contained in the customer orders is transmitted to dealers and reflects in the dealers' quoted prices.

Customer order flow received by individual dealers firstly brings dispersed information into the market. This flow is a source of private information, and represents information heterogeneity across dealers in the market. However, customer order flow is not a channel through which information is inscribed into the spot exchange rate, the interdealer order flow is. Individual dealers use this private information to trade in the interdealer market. Information from their customer orders is aggregated into interdealer order flows, and spreads across the market. This process is known as information aggregation. Dispersed information is embeded into dealer quotes once the process of

information aggregation is complete. The slow pace of information aggregation may explain the disconnect between exchange rate and fundamental over short horizons (Evans, 2005).

Evans and Lyons (2005) find evidence that the micro-based model outperforms both random walk and the macro model. They conclude that there exists nonpublic information which helps forecasting exchange rates. In their portfolio-shift model, not only public information, e.g., interest differentials, but also private information as conveyed through order flows jointly determine the exchange rate (Evans and Lyons, 2002). Other researchers have also substantiated the importance of private information. For example, Lyons (1995) and Rime (2000) demonstrate that trading flows associated with private information have an impact on prices.

1.7.3 Demand for Foreign Bond and Order Flow

In this section, I will firstly derive the demand for foreign bond for any investor in group i and j . Then each group order flow and aggregate order flow are analyzed. From equation (1.6), I can rewrite the demand for foreign bond for any investor in each group as

$$\begin{aligned}
b_{F,t}^i &= \frac{1}{\gamma\sigma_t^2}[\tilde{\lambda}_0 a_1 + \tilde{\lambda}_1 c_1 + \tilde{\gamma}_{-1} k_1 + \tilde{\gamma}_{-2} g_1]v_t^i + \frac{1}{\gamma\sigma_t^2}[\tilde{\lambda}_0 a_2 + \tilde{\lambda}_1 c_2 + \tilde{\gamma}_{-1} k_2 + \tilde{\gamma}_{-2} g_2]v_{t-1}^i - b_t^i \\
&+ \frac{1}{\gamma\sigma_t^2}[\tilde{\lambda}_0 a_3 + \tilde{\lambda}_1 c_3 + \tilde{\gamma}_{-1} k_3 + \tilde{\gamma}_{-2} g_3 - (1 + \alpha)/\alpha]s_t \\
&+ \frac{1}{\gamma\sigma_t^2}[\tilde{\lambda}_0 a_4 + \tilde{\lambda}_1 c_4 + \tilde{\gamma}_{-1} k_4 + \tilde{\gamma}_{-2} g_4 + \tilde{\gamma}_{-2} \rho_b]b_{t-2} \\
&+ \frac{1}{\gamma\sigma_t^2}[\tilde{\lambda}_0 \rho_f + \tilde{\lambda}_0 a_5 + \tilde{\lambda}_1 c_5 + \tilde{\gamma}_{-1} k_5 + \tilde{\gamma}_{-2} g_5 + (1/\alpha)]f_t \\
&+ \frac{1}{\gamma\sigma_t^2}[\tilde{\lambda}_0 a_6 + \tilde{\lambda}_1 c_6 + \tilde{\gamma}_{-1} k_6 + \tilde{\gamma}_{-2} g_6]\hat{s}_{t-1},
\end{aligned} \tag{1.14}$$

$$\begin{aligned}
b_{F,t}^j &= \frac{1}{\gamma\sigma_t^2}[\tilde{\lambda}_0 d_1 + \tilde{\lambda}_1 e_1 + \tilde{\gamma}_{-1} m_1 + \tilde{\gamma}_{-2} h_1]v_t^j - b_t^j \\
&+ \frac{1}{\gamma\sigma_t^2}[\tilde{\lambda}_0 d_3 + \tilde{\lambda}_1 e_3 + \tilde{\gamma}_{-1} m_3 + \tilde{\gamma}_{-2} h_3 - (1 + \alpha)/\alpha]s_t \\
&+ \frac{1}{\gamma\sigma_t^2}[\tilde{\lambda}_0 d_4 + \tilde{\lambda}_1 e_4 + \tilde{\gamma}_{-1} m_4 + \tilde{\gamma}_{-2} h_4 + \tilde{\gamma}_{-2}\rho_b]b_{t-2} \\
&+ \frac{1}{\gamma\sigma_t^2}[\tilde{\lambda}_0 \rho_f + \tilde{\lambda}_0 d_5 + \tilde{\lambda}_1 e_5 + \tilde{\gamma}_{-1} m_5 + \tilde{\gamma}_{-2} h_5 + (1/\alpha)]f_t \\
&+ \frac{1}{\gamma\sigma_t^2}[\tilde{\lambda}_0 d_6 + \tilde{\lambda}_1 e_6 + \tilde{\gamma}_{-1} m_6 + \tilde{\gamma}_{-2} h_6]\hat{s}_{t-1}.
\end{aligned} \tag{1.15}$$

Aggregating the demand for foreign bond for any investor over the continuum of investors in each group, this yields the aggregate demand for foreign of group i , and j investors, denoted by $B_{F,t}^i$ and $B_{F,t}^j$. The following equations write the aggregate demand for foreign bond of group i and j investors:

$$\begin{aligned}
B_{F,t}^i &= \frac{\omega}{\gamma\sigma_t^2} \left\{ [\tilde{\lambda}_0 a_1 + \tilde{\lambda}_1 c_1 + \tilde{\gamma}_{-1} k_1 + \tilde{\gamma}_{-2} g_1]f_{t+2} + [\tilde{\lambda}_0 a_2 + \tilde{\lambda}_1 c_2 + \tilde{\gamma}_{-1} k_2 + \tilde{\gamma}_{-2} g_2]f_{t+1} \right. \\
&+ [\tilde{\lambda}_0 \rho_f + \tilde{\lambda}_0 a_5 + \tilde{\lambda}_1 c_5 + \tilde{\gamma}_{-1} k_5 + \tilde{\gamma}_{-2} g_5 + (1/\alpha)]f_t \\
&+ [\tilde{\lambda}_0 a_3 + \tilde{\lambda}_1 c_3 + \tilde{\gamma}_{-1} k_3 + \tilde{\gamma}_{-2} g_3 - (1 + \alpha)/\alpha]s_t \\
&+ [\tilde{\lambda}_0 a_6 + \tilde{\lambda}_1 c_6 + \tilde{\gamma}_{-1} k_6 + \tilde{\gamma}_{-2} g_6]\hat{s}_{t-1} \\
&\left. + [\tilde{\lambda}_0 a_4 + \tilde{\lambda}_1 c_4 + \tilde{\gamma}_{-1} k_4 + \tilde{\gamma}_{-2} g_4 + \tilde{\gamma}_{-2}\rho_b]b_{t-2} - \gamma\sigma_t^2 b_t \right\},
\end{aligned} \tag{1.16}$$

$$\begin{aligned}
B_{F,t}^j &= \frac{(1 - \omega)}{\gamma\sigma_t^2} \left\{ [\tilde{\lambda}_0 d_1 + \tilde{\lambda}_1 e_1 + \tilde{\gamma}_{-1} m_1 + \tilde{\gamma}_{-2} h_1]f_{t+1} \right. \\
&+ [\tilde{\lambda}_0 \rho_f + \tilde{\lambda}_0 d_5 + \tilde{\lambda}_1 e_5 + \tilde{\gamma}_{-1} m_5 + \tilde{\gamma}_{-2} h_5 + (1/\alpha)]f_t \\
&+ [\tilde{\lambda}_0 d_3 + \tilde{\lambda}_1 e_3 + \tilde{\gamma}_{-1} m_3 + \tilde{\gamma}_{-2} h_3 - (1 + \alpha)/\alpha]s_t + [\tilde{\lambda}_0 d_6 + \tilde{\lambda}_1 e_6 + \tilde{\gamma}_{-1} m_6 + \tilde{\gamma}_{-2} h_6]\hat{s}_{t-1} \\
&\left. [\tilde{\lambda}_0 d_4 + \tilde{\lambda}_1 e_4 + \tilde{\gamma}_{-1} m_4 + \tilde{\gamma}_{-2} h_4 + \tilde{\gamma}_{-2}\rho_b]b_{t-2} - \gamma\sigma_t^2 b_t \right\}.
\end{aligned} \tag{1.17}$$

Figures 1.32–1.35 display the effect of f-shock and b-shock on currency demands of different investors.³ A positive f-shock causes an increase about 0.0038 in the aggregate foreign bond demand for group i investor. However, the same shock causes a decrease, with the same magnitude, in the aggregate foreign demand for group j investor. A negative b-shock decreases the aggregate foreign bond demand for group i investors by -0.00017 while it increases that of group j investors.

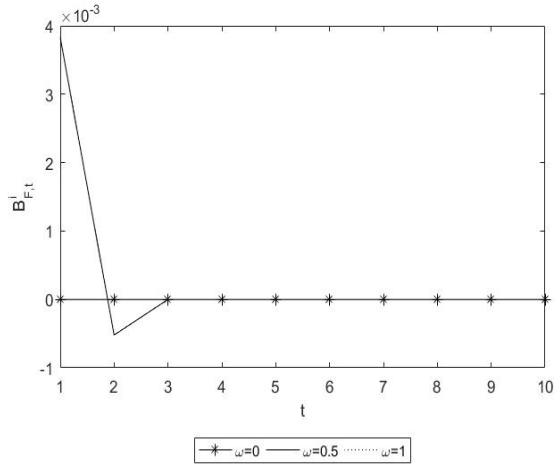


Figure 1.32: F-Shock on $B_{F,t}^i$

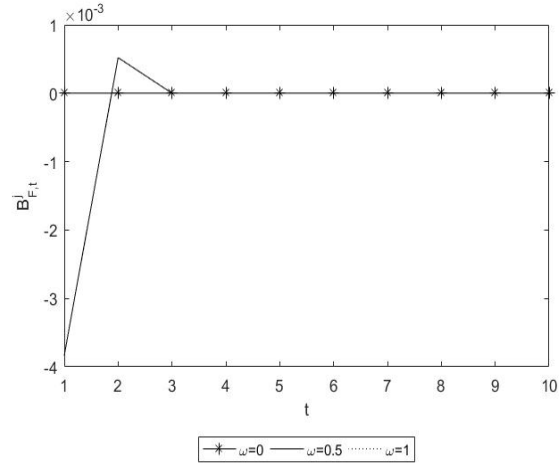
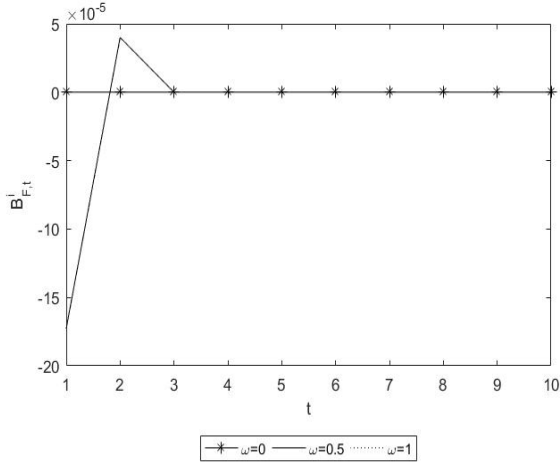
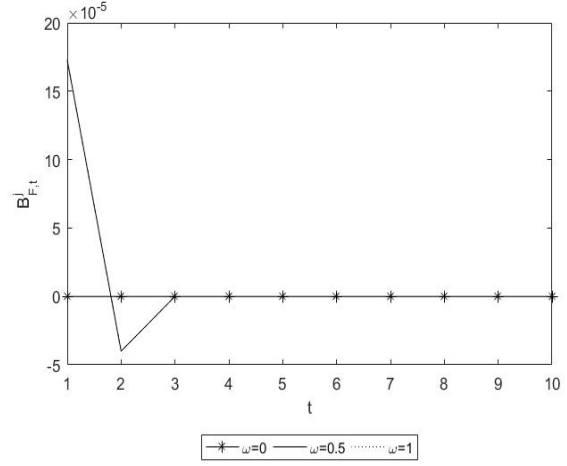


Figure 1.33: F-Shock on $B_{F,t}^j$

The demand for foreign bond is composed of three components which are the private information, the public information, and the exchange rate. Order flow captures the pure private information component in foreign bond demand. From equation (1.14) and (1.15), I aggregate the order flow over a continuum of investors in group i and j . The aggregate order flow of each group can be written as follows:

³This is for the case $\rho_b = \rho_f = 0.8$

Figure 1.34: B-Shock on $B_{F,t}^i$ Figure 1.35: B-Shock on $B_{F,t}^j$

$$\begin{aligned} \Delta x_t^i &= \frac{\omega}{\gamma \sigma_t^2} \left\{ [\tilde{\lambda}_0 a_1 + \tilde{\lambda}_1 c_1 + \tilde{\gamma}_{-1} k_1 + \tilde{\gamma}_{-2} g_1] f_{t+2} + [\tilde{\lambda}_0 a_2 + \tilde{\lambda}_1 c_2 + \tilde{\gamma}_{-1} k_2 + \tilde{\gamma}_{-2} g_2] f_{t+1} \right. \\ &\quad - [[\tilde{\lambda}_0 a_1 + \tilde{\lambda}_1 c_1 + \tilde{\gamma}_{-1} k_1 + \tilde{\gamma}_{-2} g_1] \rho_f^2 + [\tilde{\lambda}_0 a_2 + \tilde{\lambda}_1 c_2 + \tilde{\gamma}_{-1} k_2 + \tilde{\gamma}_{-2} g_2] \rho_f] f_t \\ &\quad - \gamma \sigma_t^2 (b_t - \rho_b^2 b_{t-2}) - \left. \left[[(\tilde{\lambda}_0 a_1 + \tilde{\lambda}_1 c_1 + \tilde{\gamma}_{-1} k_1 + \tilde{\gamma}_{-2} g_1) \rho_f \right. \right. \\ &\quad \left. \left. + (\tilde{\lambda}_0 a_2 + \tilde{\lambda}_1 c_2 + \tilde{\gamma}_{-1} k_2 + \tilde{\gamma}_{-2} g_2)] \frac{\tilde{\lambda}_2 \psi}{\beta_f + \tilde{\lambda}_2^2 \psi} - \frac{\rho_b \beta_f \gamma \sigma_t^2}{\tilde{\gamma}_0 (\beta_f + \tilde{\lambda}_2^2 \psi)} \right] \hat{s}_{t-1} \right\} \\ \Delta x_t^j &= \frac{(1-\omega)}{\gamma \sigma_t^2} \left\{ [\tilde{\lambda}_0 d_1 + \tilde{\lambda}_1 e_1 + \tilde{\gamma}_{-1} m_1 + \tilde{\gamma}_{-2} h_1] f_{t+1} - [\tilde{\lambda}_0 d_1 + \tilde{\lambda}_1 e_1 + \tilde{\gamma}_{-1} m_1 + \tilde{\gamma}_{-2} h_1] \rho_f f_t \right. \\ &\quad \left. - \gamma \sigma_t^2 (b_t - \rho_b^2 b_{t-2}) - \left[[\tilde{\lambda}_0 d_1 + \tilde{\lambda}_1 e_1 + \tilde{\gamma}_{-1} m_1 + \tilde{\gamma}_{-2} h_1] \frac{\tilde{\lambda}_2 \psi}{\beta_f + \tilde{\lambda}_2^2 \psi} - \frac{\rho_b \beta_f \gamma \sigma_t^2}{\tilde{\gamma}_0 (\beta_f + \tilde{\lambda}_2^2 \psi)} \right] \hat{s}_{t-1} \right\}. \end{aligned}$$

The aggregate order flow, denoted as Δx_t is the sum of Δx_t^i and Δx_t^j . The contribution in order flow is from private information $(v_t^i, v_{t-1}^i, v_t^j)$, hedge trade (b_t^i, b_t^j) , and exchange rate (\hat{s}_{t-1}) . Figures 1.36 and 1.37 display the effect of f-shock and b-shock on the aggregate flow, when $\rho_b = \rho_f = 0$. The impact of f-shock on the aggregate flow is transmitted through f-fundamental up to time $t+2$, when $\omega = 1$. When $\omega = 0$, only the shock is through f_t and f_{t+1} . When $\omega = 1$, the order flow is most affected by the change in f-shock at the first period. The effect later decreases. In period 2,

this flow has the least fluctuation. The b-shock triggers the effect on the flow not only through b_t and b_{t-2} but the exchange rate signal, \hat{s}_{t-1} . The impact of b-shock depends significantly on the coefficient of the exchange rate signal. However, the coefficient is the same when $\rho_b = \rho_f = 0$.

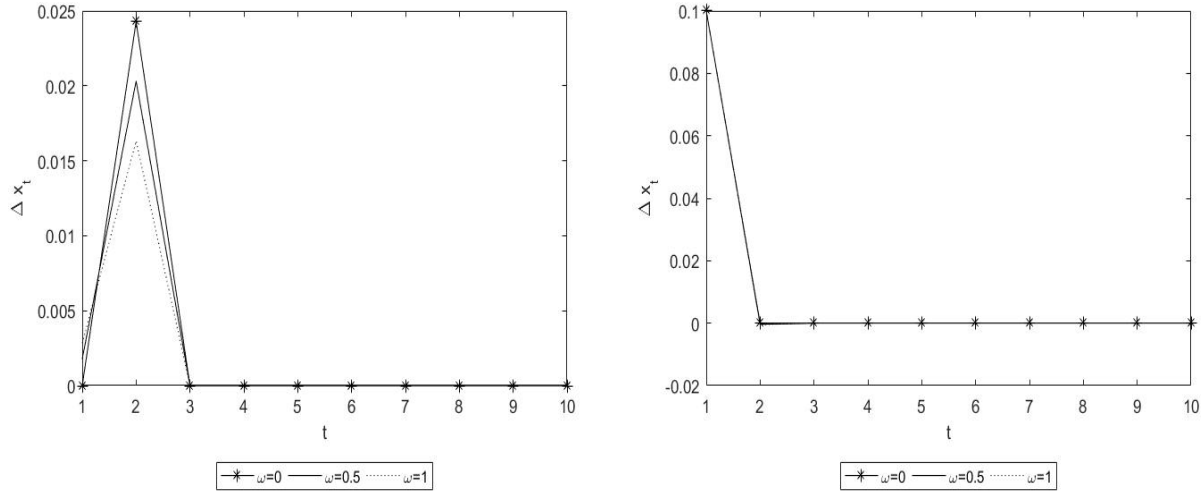


Figure 1.36: F-Shock on Aggregate Order Flow Figure 1.37: B-Shock on Aggregate Order Flow

1.7.4 Relationship between Exchange Rate and Order Flow

Taking the aggregation of the investors' demand for foreign bond and imposing market equilibrium, I can write the relationship between the exchange rate and order flow as

$$s_t = \tilde{\lambda}_0 f_t + \tilde{\gamma}_{-2} b_{t-2} + \left[\frac{\tau \beta_f + \rho_b \beta_f + \tilde{\lambda}_1 \tilde{\lambda}_2 \psi}{\beta_f + \tilde{\lambda}_2^2 \psi} \right] \hat{s}_{t-1} - \tilde{\gamma}_0 \Delta x_t. \quad (1.18)$$

From the above equation, the exchange rate has a relationship with fundamentals f_t and b_{t-2} , exchange rate signal \hat{s}_{t-1} , and order flow Δx_t . Private information about future fundamentals reaches the market and is aggregated into exchange rate via order flow. So order flow will have an impact on exchange rate. Until the future fundamentals are observed, there is also a link between exchange rate and observed fundamental.

Figures 1.38–1.41 display the R^2 of the three regressions in various degree of the fundamentals persistence. The solid line graphs the R^2 of the regression of the exchange rate on the observable fundamental. As seen in Figure 1.38 and 1.39, the R^2 is small. The R^2 of the regression of the exchange rate on the order flow, plotted by the dotted line, is larger. When taking into account of private information as proxied by the order flow, it suggests a stronger relationship between the exchange rate and the order flow. However, when f-fundamental is more persistent, the R^2 of the regression of the exchange rate on the observable fundamental is greater than the R^2 of the regression of the exchange rate and the order flow.

The R^2 of the regression of the exchange rate on both the observable fundamental and order flow, represented by the dash-dot line, is very closed to one. This suggests that including both the traditional fundamental and the order flow in the regression of the exchange rate will raise the R^2 and lessen the disconnect problem. The exchange rate has more connection with the observable fundamental when the order flow is included in the regression.

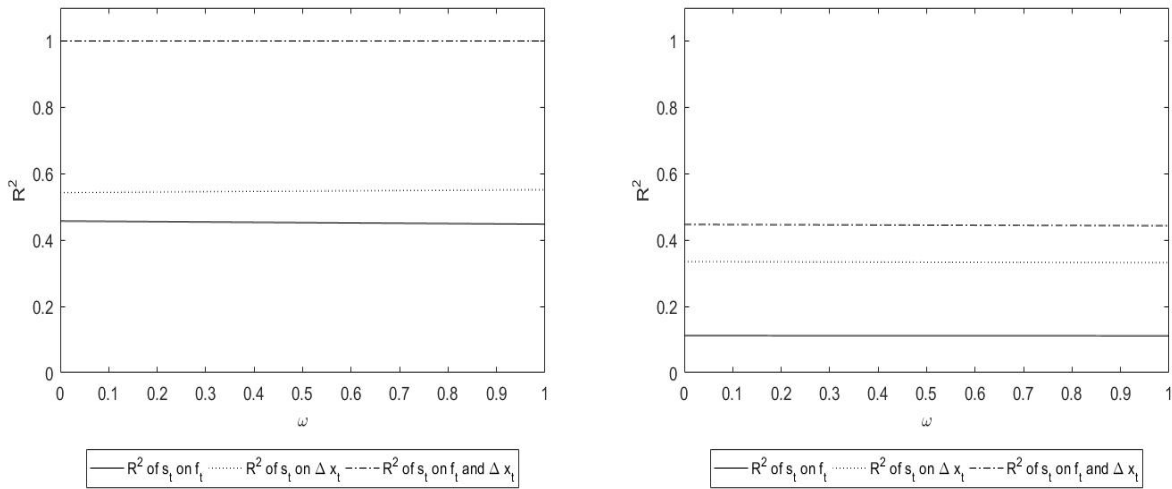


Figure 1.38: R^2 on Order Flow, $\rho_b = \rho_f = 0$ Figure 1.39: R^2 on Order Flow, $\rho_b = 0.8$ $\rho_f = 0$

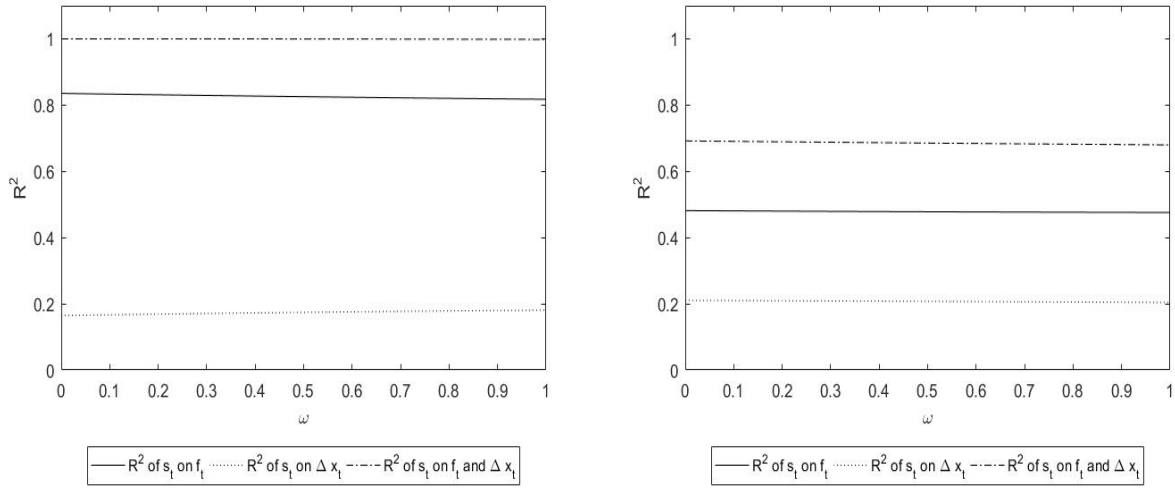


Figure 1.40: R^2 on Order Flow, $\rho_b = 0$ $\rho_f = 0.8$ Figure 1.41: R^2 on Order Flow, $\rho_b = \rho_f = 0.8$

1.8 Conclusion

In this chapter I examine the role of heterogeneous informational horizons between investors in the economy and the relationship between the exchange rate and fundamentals. The exchange rate disconnect problem is explained by introducing information heterogeneity into a dynamic monetary model of exchange rate determination. The model addresses the importance of the information asymmetry among investors in the economy.

I find that a differential informational horizon across investors causes the exchange rate to disconnect with the current observed fundamental. The intensity of the disconnect problem is greater when extending the model from one-period-model to differential informational horizon model, and to two-period model. That is, the disconnect between the exchange rate and the current observable fundamental arises as the fraction of better-informed investors increases. The reason is simple. When there are more investors who know information about future fundamentals at a long period ahead, the existence of the differential among private information increases, which stresses the significance of the unobservable components in the model. Thus it deteriorates the relationship

between the exchange rate and the current observable fundamental. Knowing more about future fundamentals would not help the exchange rate connect with the current observable fundamental unless the process of fundamental f_t is more persistent. Besides, the heterogeneous information also causes the exchange rate to be more volatile. An increase in the fraction of investors who receive information about future fundamentals at a long period ahead creates the fluctuation in the currency market. The paper also explores the relationship between the exchange rate and the order flow. I find that the order flow can explain a variation in the exchange rate better than the observable fundamental. The study of exchange rate determination should incorporate the order flow as one of the variables related to exchange rate.

1.9 References

- Allen, Franklin; Morris, Stephen and Shin, Hyun Song. "Beauty Contests and Iterated Expectation in Asset Markets." *Review of Financial Studies*, 2006, 19(3), pp. 719-752.
- Bacchetta, Philippe and van Wincoop, Eric. "Can Information Heterogeneity Explain the Exchange Rate Determination Puzzle?" *American Economic Review*, 2006, 96(3), pp.552-576.
- Bjønnes, Geir and Dagfinn, Rime. "Dealer Behavior and Trading Systems in Foreign Exchange Markets." *Journal of Financial Economics*, 2005, 75, pp. 571-605.
- Evans, Martin D.D. "Foreign Exchange Market Microstructure." Working Papers, 2005, Georgetown University, Department of Economics.
- Evans, Martin D.D. and Lyons, Richard K. "Order Flow and Exchange Rate Dynamics." *Journal of Political Economy*, 2002, 110, pp. 170-180.
- Evans, Martin D.D. and Lyons, Richard K. "Meese-Rogoff Redux: Micro-Based Exchange-Rate Forecasting." *American Economic Reviews Papers and Proceedings*, 2005, 95(2), pp. 405-414.
- Grauwe, De. P. and Grimaldi, Marianna. "Heterogeneity of Agents, Transaction Costs and Exchange Rate." *Journal of Economic Dynamics and Control*, 2005, 29, pp. 691-719.
- Greene, William H. *Econometric Analysis (7th edition)*, England:Pearson Education, 2012
- Lyons, Richard K. "Tests of Microstructural Hypothesis in the Foreign Exchange Market." *Journal of Financial Economics*, 1995, 39, pp. 321-351.
- Madhavan, Ananth N. and Seymour, Smidt. "An Analysis of Changes in Specialist Inventories and Quotations." *Journal of Finance*, 1993, 48, pp. 1595-1628.

- Mark, Nelson C. "Exchange Rates and Fundamentals: Evidence on Long-Horizon Predictability." *American Economic Review*, 1995, 85(1), pp. 201-18.
- Meese, Richard A. and Rogoff, Kenneth. "Empirical Exchange Rate Models of the Seventies: Do They Fit out of Sample?", *Journal of International Economics*, 1983, 14(1-2), pp. 3-24.
- Rime, Dagfinn. "Private or Public Information in the Foreign Exchange Markets? An Empirical Analysis", Memorandum 14/2000, Department of Economics, University of Oslo.

CHAPTER 2. REAL EXCHANGE RATES, EXTERNAL IMBALANCES, AND CURRENCY RISK PREMIA

The violation of uncovered interest parity suggests that interest rate differentials can predict currency excess returns. I investigate whether there are variables other than interest rate differentials explaining these excess returns. I combine a present-value model that relies on real exchange rates to explain currency returns with a portfolio balance model that focuses on the impact of a country's external imbalances on excess returns. I first present a theoretical model that is then tested with data on real exchange rates and international investment positions of a set of developed countries. I find that, apart from interest rate differentials, real exchange rates and external imbalances are important factors in determining currency excess returns. Adding real exchange rates alone or external imbalances alone does not statistically improve the ability of the model to forecast currency excess returns using interest rate differentials. Both variables must be added for the new model to outperform the traditional one.

2.1 Introduction

The uncovered interest parity (UIP) condition states that the expected return on domestic deposits will equal the exchange rate-adjusted expected return on foreign deposits. That is, under perfect capital mobility, investors will be indifferent to return on deposits between the two countries. By definition, the excess return on foreign deposits is the sum of the difference between the interest in two countries and the percentage change in the exchange rate over the period of investment. If UIP holds, the high interest rate currency is required to depreciate to offset the interest rate differential.

UIP is a natural equilibrium outcome in the standard models of exchange rate determination under flexible exchange rates and perfect capital mobility. Empirically, however, exchange rates

are found to be uncorrelated with macroeconomic variables, namely, interest rate, money supply, and inflation. That is, these macro variables are not helpful in forecasting exchange rates. Engel and West (2005) find that the exchange rates follow a random walk. Obstfeld and Rogoff (2001) coin the term ‘exchange rate disconnect’ to designate the lack of correlation between the exchange rate and the relevant macro variables. More broadly, the exchange rate disconnect includes many puzzles in international macroeconomics, e.g., the purchasing power parity puzzle, the Backus and Smith puzzle, and the UIP puzzle. The UIP puzzle relates to the fact that exchange rates of high-interest-rate countries depreciate insufficiently to offset interest rate differentials; on the contrary, currencies appreciate in response to a rise in the interest rate.

The deviation from UIP, tested by Fama (1984), suggests that there are potential gains from an increase in interest rate. That is, the currency excess returns would not be zero as predicted by the UIP. This expected excess return can be thought of as an exchange rate risk premium. The positive excess returns are the compensation for time-varying risks. Menkhoff et al. (2012) show that the deviation from UIP can be accounted for a compensation for risk. In order to resolve UIP puzzle, researchers have investigated the driving force of this positive currency return. For example, Backus et al. (2001) relate the factors driving the foreign exchange risk premium to the stochastic discount factors. Verdelhan (2010), Bansal and Shaliastovich (2013), and Colacito and Croce (2013) relate a risk premium to the variances of consumption in the home and foreign countries. A positive excess return compensates an investor for taking on consumption growth risk. However, the recent literature is still silent about the economic determinants underlying those excess returns.

In this study, I study predictors of currency excess returns by investigating the relationship between the currency risk premium, the interest rate differential, the real exchange rate, and the net foreign asset position. The conceptual framework is based on the present-value of the real exchange rate. If purchasing power parity (PPP) fails to hold in the long-run, expected excess returns are related by the expected real interest rate differential, the real exchange rate, and macroeconomic fundamentals that influence the long-run mean of the real exchange rate.

Let $q_t \equiv s_t + p_t^* - p_t$, where q_t is the log real exchange rate, s_t is the log nominal exchange rate, p_t^* and p_t are the log of the foreign and the domestic price levels, respectively. The excess returns on foreign deposits, denoted by $\rho_{t+1} = s_{t+1} - s_t + i_t^* - i_t$, where i_t and i_t^* are nominal interest rates of the domestic and the foreign countries, respectively, can be expressed in terms of real variables as:

$$\rho_{t+1} = q_{t+1} - q_t + (i_t^* - \pi_{t+1}^*) - (i_t - \pi_{t+1}),$$

where $\pi_{t+1} \equiv p_{t+1} - p_t$ and $\pi_{t+1}^* \equiv p_{t+1}^* - p_t^*$ are the domestic and foreign inflation rates. Rewrite the above equation in terms of the real interest rates, denoted by r_t for the domestic country and r_t^* for the foreign country, iterate forward, and take conditional expectations to get

$$q_t - \lim_{k \rightarrow \infty} E_t(q_{t+k}) = \sum_{j=1}^{\infty} E_t(r_{t+j-1}^* - r_{t+j-1}) - \sum_{j=1}^{\infty} E_t \rho_{t+j}.$$

When PPP is assumed to hold in the long run, the real exchange rate is stationary, and $\lim_{k \rightarrow \infty} E_t(q_{t+k})$ is the unconditional mean of the real exchange rate. Fluctuations in the real exchange rate are mainly affected only by the infinite sum of the deviations from real rate equality (interest differential) and the deviations from UIP (risk premium). Assuming PPP holds, Balduzzi and Chiang (2017), and Dahlquist and Penasse (2016) show evidence that apart from the interest rate differential, the real exchange rate is an important driver of the currency risk premium. If PPP fails to hold, the expected currency risk premium depends on three main driving forces. The first force captures movements of the real exchange rate. The second force captures an infinite cumulative of real interest rate differentials. The third force includes traditional macro variables (fundamentals) that are the driving force underlying the expectation of long-run real exchange rates. Menkhoff et al. (2017) control for the influence of fundamentals on the long-run real exchange rate to sharpen the relationship between real exchange rates and currency risk premia. Adjusting the real exchange rates for key country-specific fundamentals, namely, productivity (to capture Harrod-Balassa-Samuelson effects that countries with higher productivity in the tradable sector experience stronger in real exchange rates), export quality (countries with higher quality of export goods experience stronger real exchange rates), net foreign assets (currency of countries with

net foreign asset deficits have the real exchange rate depreciated), and output gaps (central banks follow the Taylor rule linking the real exchange rate to output gaps) better isolates information related to the currency excess returns. Moreover, using the real exchange rates and adjusting for the fundamentals as a portfolio signal deliver a precise measure of the currency value. As a result, this contributes to a strong predictive power for the future currency returns.

The present-value perspective relates currency excess returns with interest rate differentials, real exchange rates, and fundamentals that evolve with time and influence in the long-run real exchange rate. In this chapter, I study determinants of currency excess returns by proposing another two factors, in addition to interest rate differentials, for expected currency returns. I focus on the role of real exchange rates as well as external imbalances in influencing currency risk. I consider external imbalances, measured by a country's net foreign assets, to be a proxy for fundamentals relevant to the long-run real exchange rate. I find evidence of UIP violations where high interest rate currencies provide positive currency excess returns. In addition, both real exchange rates and external imbalances are also important factors in determining currency returns. When the real exchange rate is high, the excess return on foreign deposits will be low because the foreign currency is expensive relative to the home currency. Besides, a country with negative external imbalances will offer positive currency excess returns. A net debtor country provides a positive currency risk premium in order to compensate investors who are willing to finance negative external imbalances. Moreover, the best model for forecasting future currency excess returns is the model with both real exchange rates and external imbalances, together with interest rate differentials.

The rest of the chapter is organized as follows. Section 2.2 presents a literature review about the UIP puzzle, and the link of currency returns with real exchange rates and external imbalances. The theoretical model is proposed in section 2.3. In section 2.4, I discuss the method of estimation, the data used in the study, and empirical results. Model forecasting is also shown in this section. Lastly, the conclusions of the study are provided in section 2.5.

2.2 The Link between the Real Exchange Rates and External Imbalances, and the UIP Puzzle

In this section, I explain the theory behind the relations between the deviation from UIP and the real exchange rates, and the relations between the external imbalances and the currency excess returns.

2.2.1 The UIP Puzzle

Let S_t be the exchange rate in terms of home currency per one unit of foreign currency. The return on a home deposit with an investment of 1 unit of home currency is $1 + i_t$. The return on a foreign deposit with the same amount of money invested is $(1 + i_t^*) \frac{S_{t+1}}{S_t}$. Under log-linearization, the UIP condition can be expressed as

$$E_t(s_{t+1} - s_t + i_t^* - i_t) = 0.$$

This states that the nominal exchange rate is expected to move in order to compensate for any difference in the interest rate, and the currency risk premium should be zero.

The expected excess return on foreign deposits is defined as $E_t \rho_{t+1} \equiv E_t(s_{t+1} - s_t + i_t^* - i_t)$. A deviation from UIP implies that $E_t \rho_{t+1} \neq 0$. Fama (1984) tested the validity of UIP by estimating the following regression:

$$s_{t+1} - s_t = a + \tilde{b}(i_t - i_t^*) + u_{t+1}.$$

Under the null hypothesis of UIP, the regression coefficients should be $a = 0$ and $\tilde{b} = 1$. Using the definition for currency risk premia, we can rewrite this regression as

$$\rho_{t+1} = a + b(i_t^* - i_t) + u_{t+1},$$

where $b = 1 - \tilde{b}$. If UIP holds, then $E_t(\rho_{t+1}) = 0$ and $cov(\rho_{t+1}, x_t) = 0$ for any variable x_t , e.g. $(i_t^* - i_t)$. Hence, there should be no variable that can forecast excess returns. However, the estimation results are such that the estimated value of b is greater than zero.¹ There is a positive

¹A body of empirical works has found the estimation of \tilde{b} to be less than one, usually less than zero. This leads to the violation of UIP, and implies that the currency with high interest rate tends to appreciate, which contrasts with the UIP prediction that the high interest rate currency will depreciate.

covariance between the excess return on foreign deposits and the foreign-home interest differential, $cov(\rho_{t+1}, i_t^* - i_t) > 0$. Hence, the high interest rate currency is expected to have a positive excess return.²

Since a currency risk premia exists, investors earn profits by arbitraging the difference between exchange rates and interest rates in those countries. Investors may follow the investment rule called *carry trade* by taking a long position in the currency with the higher interest rate. If foreign interest rates are higher than home interest rates, then foreign deposits have excess returns over home deposits, and vice versa. Many studies, for example, Burnside et al. (2008, 2011), Lustig and Verdelhan (2007), Brunnermeier et al. (2009), and Lustig et al. (2011) have found evidence of carry trade. There is a high return with a low standard deviation in taking long positions in high-interest rate countries and short positions in low-interest rate countries.

Engel (2016) and Valchev (2015), however, find that the relationship between currency excess returns and interest rate differentials changes over long horizons. At a short horizon, high interest rates predict positive currency excess returns, which is $cov(E_t \rho_{t+1}, i_t^* - i_t) > 0$. But at longer horizons, high interest rates tend to predict negative excess returns, $cov(E_t \rho_{t+j}, i_t^* - i_t) < 0$ for some $j > 0$. Valchev (2015) finds that the currency with high interest rate today forecasts positive currency excess returns at horizons of up to 3 years, but it forecasts negative excess returns at horizons of 4 to 7 years. This finding on longer horizons also violates the UIP but in the opposite direction. Engel (2016) reasons that there is excess comovement in the level of the exchange rate and the interest differential. The covariance of the stationary component of the exchange rate with the foreign-home interest differential is more negative than would hold under interest parity: $cov(E_t \sum_{j=0}^{\infty} \rho_{t+j+1}, i_t^* - i_t) < 0$. Since we have $cov(E_t \rho_{t+1}, i_t^* - i_t) > 0$ and $cov(E_t \sum_{j=0}^{\infty} \rho_{t+j+1}, i_t^* - i_t) < 0$, there must be the case that $cov(E_t \rho_{t+j}, i_t^* - i_t) < 0$ for some $j > 0$.

²See, for example, Fama (1984), Engel (1996), Froot and Thaler (2001), and Burnside et al. (2006)

2.2.2 Currency Risk Premia and Real Exchange Rates

The reverse relationship between interest rate differentials and currency risk premia over time suggests that interest rate differentials alone cannot describe the time variation in currency risk premia. Dahlquist and Penasse (2016) argue that currency excess returns depending only on interest differentials cannot explain the fluctuations in real exchange rates. To quantitatively match the movements of real exchange rates in the data, the currency risk premia must be very volatile and depend on an additional component which is related to the real exchange rate. While UIP violations link interest rate differentials to currency returns, PPP relates the real exchange rate to the infinite sum of future currency returns.

Assume that $\lim_{k \rightarrow \infty} E_t(q_{t+k}) = \mu_q$. If currency risk premia are well described by the Fama regression, with the additional assumptions that the interest differential follows an AR(1) process and the inflation differential is unpredictable, the real exchange rate will be negatively correlated with the current interest rate differential, $q_t - \mu_q = \tilde{b} \frac{i_t^* - i_t - \mu_i}{1 - \rho_i}$ where μ_i and ρ_i are AR(1) parameters; and \tilde{b} is the estimate coefficient from the Fama regression which tends to be negative, and the R^2 should be high. However, they find that the coefficients of the regression of the real exchange rates on interest rate differentials for many currencies in the sample are positive, suggesting that real exchange rates appreciate with interest rate differentials, and R^2 values are small. Therefore, the simple model in which the risk premium depends only on the difference between foreign and domestic interest rates might not be correct.

Dahlquist and Penasse (2016) hypothesize a two-factor structure for the expected currency returns. The first factor is the interest rate differential. The second factor relates to the long run relationship of real exchange rate. Thus, the Fama regression is augmented as:

$$\rho_{t+1} = a + b(i_t^* - i_t) + \eta_t + u_{t+1}.$$

They derive an additional component, $\eta_t = (1 - \rho_\eta) \left[\beta \frac{i_t^* - i_t - \mu_i}{1 - \rho_i} - (q_t - \mu_q) \right]$, related to the real exchange rate, where ρ_η is the persistence of η , and ρ_i and μ_i relate to the dynamics of the interest rates. Hence, the currency return predictions should incorporate not only the interest rate differential but also the real exchange rate. Including the real exchange rate, the R^2 in predictive

regressions for monthly returns increases by 30%. The predictability holds out-of-sample and increases over long horizons. Their model also accounts for the relation that the real exchange rates appreciate with the interest rate differentials, as well as the flip nature of the currency return and interest rate differential at long horizons. In addition, Balduzzi and Chiang (2017) find that the real exchange rate indeed predicts negative excess returns. They also highlight the predictive power of the real exchange rate for the currency risk premia. The real exchange rate has the strongest predictive power, followed by the interest rate differential.

2.2.3 Currency Risk Premia and External Imbalances

An increase in international financial integration has led to a shift in the composition of countries' asset and liability positions. The dynamic adjustment of a country's external balance is of central interest to international economists. The movement of wealth transfers across countries also affects fluctuations in exchange rates, which can lead to imbalances in trade and capital flows, and eventually contribute to the causes and consequences of financial crises. Understanding what determines the fluctuations in exchange rates and the adjustment process of a country's external imbalance has drawn attention to policymakers and investors in currency markets. Gourinchas and Ray (2007) show that a country's external constraint holds when today's external imbalances predict either future net export growth or future movements in net foreign asset portfolio returns, or both. The adjustment in negative external accounts implies a surplus in future net exports through the trade channel, and an increase in expected net foreign portfolio returns through the valuation channel. The exchange rate plays a vital role in this adjustment. In the short run, a depreciation raises the value of foreign assets held by the home country, causing an adjustment via the valuation channel. In the longer run, home currency depreciation favors trade surpluses, contributing to a trade channel adjustment. The international financial adjustment affects exchange rate predictability. Gourinchas and Ray (2007) empirically find that external imbalances have a strong predictive power for exchange rates, and that a deficit in external imbalances predicts a

depreciation of the domestic currency. The results hold for out-of-sample forecast and the model outperforms the random walk at all horizons between 1 and 16 quarters.

Gabaix and Maggiori (2015) propose a model of exchange rate determination based on capital flows in imperfect financial markets, called Gamma model. This kind of portfolio balance model proposes that exchange rates are determined by external imbalances and financiers' risk-bearing capacity. In their model, there are two countries. Each country borrows or lends in its own currency. Global financial firms—called financiers—absorb countries' trade imbalances. They are long in the debtor country and short in the creditor country. However, they have financial constraints that limit their ability to take positions, based on their risk-bearing capacities and existing balance sheet risks. Financiers are unable to intermediate currency mismatches, even though there are excess returns, because they face limits in their risk bearing capacity. An important feature of the model is that a country's imbalance is a driver of the currency risk premia. Define the returns of the carry trade as $R^c \equiv \frac{R^* S_1}{R S_0} - 1$, where R and R^* are return on domestic and foreign assets, S_t is the nominal exchange rate in period $t = 0, 1$. They derive the expected currency excess returns as follows:

$$E(R^c) = \Gamma \frac{\frac{R^*}{R} E(imp_1) - imp_0}{(R^* + \Gamma) imp_0 + \frac{R^*}{R} E(imp_1)},$$

where Γ represents risk-bearing capacity of financiers, imp_t is the home import value in terms of home currency at time t , with exports normalized to unity. Substituting the equilibrium exchange rate, $E(imp_1) - imp_0$ can be derived in the model as the home country's net exports. In a two-country, two-period model, there is a positive relation between the evolution of net exports and net foreign assets. The currency risk premia will be higher when (i) the return differential is larger, (ii) the funding country is a net foreign creditor, and (iii) finance is more imperfect (higher Γ).

The intuition of the influence of external imbalances on currency risk premia is that investors require a risk premium in order to hold the currency of net debtor countries. Della Corte et al. (2016) find evidence that a global imbalance risk factor has pricing power in currency risk premia. In addition to interest rate differentials, currency excess returns are driven by the external debt and its currency denomination. The debtor country is considered riskier, and is likely to

issue liabilities in foreign currency due to, for instance, its political instability and inflation risk. Hence, currency excess returns are higher for net debtor countries with higher propensity to issue liabilities in foreign currency. Moreover, they also show that net foreign asset positions contain related but not identical information to interest differentials in the cross-section of currencies. The riskiest countries in terms of net foreign asset positions are not necessarily the highest interest rates countries; hence, the impact of net foreign asset positions on currency risk premia differ from the interest rate channel.

In this study, the two ideas of the role of real exchange rates and external imbalances in affecting currency risk premia are blended together. I develop a benchmark model and estimate its various versions for predicting currency premia. More specifically, I consider a standard Fama regression model, a regression with interest rate differentials and real exchange rates, and a regression that includes the previous factors and in addition accounts for external imbalances. The currency risk premia model that I develop below utilizes a version of the model developed by Itskhoki and Mukhin (2017). The theoretical model is discussed in the next section.

2.3 The Model

In this section, I firstly present a theoretical model that explains the importance of the real exchange rate and the external imbalance on currency risk premium. Then, a model for estimation is derived from the theoretical part. The model closely follows Itskhoki and Mukhin (2017). They propose a dynamic general equilibrium model of exchange rate disconnect in which the disconnect is generated by an exogenous shock to international asset demand (or a financial shock). In their model, the deviation from UIP depends on a country's net foreign position and a financial shock.

2.3.1 The Theoretical Model

There are two countries: home and foreign (foreign country variables are denoted with a *). Each country offers a bond in its own currency. Home and foreign households trade goods internationally. The distinct feature of the model is that all international transactions are intermediated by the financial sector. A country can have a trade imbalance, and the financial sector absorbs this imbalance by taking a long position in the debtor country and a short position in the creditor country.

Consider the international financial market, which is driven by three types of agents: home and foreign households, noise traders, and arbitrageurs. Home and foreign households hold only their local-currency bond, and take net foreign positions B_{t+1} and B_{t+1}^* , respectively. There are n noise traders taking a zero-capital position. For example, they long N_{t+1}^* in foreign-currency bonds and short $N_{t+1} = -N_{t+1}^* S_t$ in home-currency bonds, and vice versa when $N_{t+1}^* < 0$. Noise traders have imperfect knowledge about the market. They rely on nonfundamental-based trading techniques. The demand for foreign bonds by noise traders is affected only by noise that is unrelated to economic fundamentals. In the paper, the noise is the financial shock (ψ). The presence of noise trading creates volatility in the market. Informed traders or competitive arbitrageurs are knowledgeable about the market. They make their decisions based on rational expectations about the future. Also, there are m arbitrageurs that take a zero-capital position long D_{t+1}^* in foreign-currency bonds and short $D_{t+1} = -D_{t+1}^* S_t$ home-currency bonds, and vice versa when $D_{t+1}^* < 0$.

Households: A representative home household maximizes the discounted expected utility:

$$E_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{1}{1-\sigma} C_t^{1-\sigma} - \frac{1}{1+1/\nu} L_t^{1+1/\nu} \right)$$

subject to the constraint:

$$P_t C_t + B_{t+1} = R_{t-1} B_t + W_t L_t + \Pi_t, \quad (2.1)$$

where C_t is the home country's composite consumption of home goods and foreign goods, household expenditure between home and foreign goods is $P_t C_t = P_{Ht} C_{Ht} + P_{Ft} C_{Ft}$, L_t is home labor supply, σ is the relative risk aversion parameter and ν is the elasticity of labor supply, P_t is the consumer

price index, B_{t+1} is nominal value of home bond purchased at time t and paying $R_t B_{t+1}$ units of home currency at time $t + 1$, W_t is the wage rate, and Π_t are firms' profits.

The households' optimal labor supply is

$$C_t^\sigma L_t^{1/\nu} = \frac{W_t}{P_t}. \quad (2.2)$$

Home households can trade only the home currency bond. Their optimal saving choice gives rise to

$$1 = R_t E_t \beta \left(\frac{C_{t+1}}{C_t} \right)^{-\sigma} \frac{P_t}{P_{t+1}}. \quad (2.3)$$

Firms: There are two inputs used in production: labor L_t , and intermediate input X_t . Output is produced by the production function:

$$Y_t = L_t^{1-\phi} X_t^\phi,$$

where ϕ is the elasticity of output with respect to intermediate inputs.

Intermediate inputs are the same bundle as consumption, hence their price index is given by P_t . The marginal cost is given by :

$$MC_t = \left(\frac{W_t}{1-\phi} \right)^{1-\phi} \left(\frac{P_t}{\phi} \right)^\phi. \quad (2.4)$$

Firms optimally allocate expenditure between labor and intermediates. Their optimal demands for labor and intermediate inputs are

$$W_t L_t = (1 - \phi) MC_t Y_t \quad \text{and} \quad P_t X_t = \phi MC_t Y_t. \quad (2.5)$$

The expenditure on X_t consists of the domestic and foreign varieties, X_{Ht} and X_{Ft} . The profits of the domestic firms consists of the profits from selling home-produced goods in domestic and foreign markets: $\Pi_t = (P_{Ht} - MC_t) Y_{Ht} + (P_{Ht}^* S_t - MC_t) Y_{Ht}^*$, where total output is allocated between the home and the foreign market, $Y_t = Y_{Ht} + Y_{Ht}^*$.

Substitution of firms profits and labor demand in the household budget constraint, equation (2.1), yields the home country budget constraint:

$$B_{t+1} - R_{t-1}B_t = NX_t,^3 \quad (2.6)$$

where $NX_t = S_t P_{Ht}^* Y_{Ht}^* - P_{Ft} Y_{Ft}$ is the home currency net exports.

Foreign Country: The foreign households are symmetric. The bond demand by foreign households is given by:

$$1 = R_t^* E_t \beta \left(\frac{C_{t+1}^*}{C_t^*} \right)^{-\sigma} \frac{P_t^*}{P_{t+1}^*}. \quad (2.7)$$

Foreign firms are also symmetric. The foreign inputs demand and prices are symmetric with their home country counterparts.

Financial Sector: For noise traders, I assume that the noise traders' demand for foreign-currency bonds is only affected by the currency demand shock, denoted by ψ_t , and given by:

$$N_{t+1}^* = n(e^{\psi_t} - 1). \quad (2.8)$$

Arbitrageurs are informed traders. They know about fundamentals in the economy. They use this knowledge in their trading decision. For example, arbitrageurs know that returns are log-normally distributed. They use this information in maximizing their mean-variance utility of excess returns.

Each arbitrageur $j = 1, \dots, m$ chooses to invest d_{t+1}^{*j} in foreign-currency bonds to maximize the expected utility of his total wealth. The maximization problem is

$$\max_{d_{t+1}^{*j}} -E_t e^{-\omega \tilde{W}_{t+1}^j},$$

where ω is the risk aversion parameter. The wealth of arbitrageur j , denoted by \tilde{W}_{t+1}^j is given by $\tilde{W}_{t+1}^j = \tilde{R}_{t+1} d_{t+1}^{*j}$, where $\tilde{R}_t \equiv R_{t-1}^* - R_{t-1} \frac{S_{t-1}}{S_t}$ is the excess return on the foreign investment in terms of foreign currency.

³The derivation uses the fact that $P_t C_t = P_{Ht} C_{Ht} + P_{Ft} C_{Ft}$, $P_t X_t = P_{Ht} X_{Ht} + P_{Ft} X_{Ft}$, and $Y_t = C_t + X_t$

I also assume that \tilde{R} is normally distributed. The above maximization problem is equivalent to maximizing the mean-variance utility of excess returns:

$$\max_{d_{t+1}^{*j}} E_t \tilde{R}_{t+1} d_{t+1}^{*j} - \frac{\omega}{2} \text{var}_t(\tilde{R}_{t+1})(d_{t+1}^{*j})^2.$$

The arbitrageur individual demand for foreign-currency bonds is

$$d_{t+1}^{*j} = \frac{E_t \tilde{R}_{t+1}}{\omega \text{var}_t(\tilde{R}_{t+1})}.$$

Therefore, total demand for foreign bonds by arbitrageurs is given by:

$$D_{t+1}^* = m \frac{E_t \tilde{R}_{t+1}}{\omega \text{var}_t(\tilde{R}_{t+1})}. \quad (2.9)$$

For concreteness, I assume that the profits and losses of the noise traders and arbitrageurs are transferred to the foreign households. Thus, the foreign budget constraint differs from the home constraint in that it adds a term $\tilde{R}_t(N_t^* + D_t^*)$ into the equation. Thus, the foreign constraint can be written as:

$$B_{t+1}^* - R_{t-1}^* B_t^* = NX_t^* + \tilde{R}_t(N_t^* + D_t^*). \quad (2.10)$$

Equilibrium System: The model is in equilibrium in the asset, goods, labor markets, and the condition of a country budget constraint. The labor market clears when labor demand in equation (2.5) equates labor supply in equation (2.2) with marginal cost in (2.4), and also its symmetric foreign counterpart. The goods markets in home and foreign country clearing require $Y_t = Y_{Ht} + Y_{Ht}^*$ and $Y_t^* = Y_{Ft} + Y_{Ft}^*$, where

$$\begin{aligned} Y_{Ht} &= C_{Ht} + X_{Ht} = (1 - \gamma)h \left(\frac{P_{Ft}}{P_t} \right) [C_t + X_t], \\ Y_{Ht}^* &= C_{Ht}^* + X_{Ht}^* = \gamma h \left(\frac{P_{Ht}^*}{P_t^*} \right) [C_t^* + X_t^*], \\ Y_{Ft} &= \gamma h \left(\frac{P_{Ft}}{P_t} \right) [C_t + X_t], \\ Y_{Ft}^* &= (1 - \gamma)h \left(\frac{P_{Ft}^*}{P_t^*} \right) [C_t^* + X_t^*]. \end{aligned}$$

Goods are produced to satisfy goods demand given the price index $P_t = (P_{Ht}C_{Ht} + P_{Ft}C_{Ft})/C_t$ and the foreign counterpart P^* , where $(P_{Ht}, P_{Ht}^*, P_{Ft}, P_{Ft}^*)$ are given by

$$\begin{aligned} P_{Ht} &= MC_t^{1-\alpha} P_t^\alpha, \\ P_{Ht}^* &= \left(\frac{MC_t}{S_t}\right)^{1-\alpha} (P_t^*)^\alpha, \\ P_{Ft} &= (MC_t^* S_t)^{1-\alpha} P_t^\alpha, \\ P_{Ft}^* &= (MC_t^*)^{1-\alpha} (P_t^*)^\alpha. \end{aligned}$$

Asset demands in equation (2.3), (2.7), (2.8), (2.9) satisfy the financial market clearing conditions, which require $B_{t+1} + N_{t+1} + D_{t+1} = 0$ and $B_{t+1}^* + N_{t+1}^* + D_{t+1}^* = 0$. Lastly, the system also requires that the home budget constraint in (2.6) and foreign counterpart in (2.10) be met.

Log-Linearized System: I consider the equilibrium system that is log-linearized around the symmetric steady state.⁴ Define a small letter as the log deviation of that variable from its steady state, for example $c_t = \log C_t - \log \bar{C}$, $p_t = \log P_t - \log \bar{P}$, and $i_t = \log R_t - \log \bar{R}$, except for $b_{t+1} \equiv \bar{R}B_{t+1}/\bar{P}\bar{Y}$ and $nx_t \equiv NX_t/\bar{P}\bar{Y}$. Since the steady state of B_{t+1} and NX_t are zero, I define b_{t+1} as a linear deviation of the net foreign asset from its steady state $\bar{B} = 0$, and nx_t as the linear deviation of net exports from steady state $\bar{NX} = 0$. The steady state exchange rate is $\bar{S} = 1$ and the steady state return is $\bar{R} = 1/\beta$.

Real Exchange Rates and Prices: The log-linear approximation for the price index P_t is $p_t = (1 - \gamma)p_{Ht} + \gamma p_{Ft}$, where γ is the home bias parameter. Also, the log linearization of P^* is $p_t^* = \gamma p_{Ht}^* + (1 - \gamma)p_{Ft}^*$. The log linearization of $(P_{Ht}, P_{Ht}^*, P_{Ft}, P_{Ft}^*)$ prices are:

$$\begin{aligned} p_{Ht} &= (1 - \alpha)(1 - \phi)(w_t - p_t) + p_t, \\ p_{Ht}^* &= (1 - \alpha)(1 - \phi)(w_t - p_t) + (1 - \alpha)p_t + \alpha p_t^*, \\ p_{Ft} &= (1 - \alpha)(1 - \phi)(w_t^* - p_t^*) + p_t^*, \\ p_{Ft}^* &= (1 - \alpha)(1 - \phi)(w_t^* - p_t^*) + (1 - \alpha)p_t^* + \alpha p_t. \end{aligned}$$

⁴The derivation of the log-linearized system is shown in the Appendix B.1.

Substitute for the prices, the log real exchange rate can be expressed in terms of the log nominal exchange rate as follows

$$q_t = \frac{1}{1 + \frac{1}{1-\phi} \frac{2\gamma}{1-2\gamma}} s_t. \quad (2.11)$$

Real Exchange Rates and Quantities: Starting from the labor supply and labor demand equations, one can derive the relationship between the consumption differential and the real exchange rate, and the relationship between the output differential and the real exchange rate as follows:

$$\begin{aligned} c_t - c_t^* &= -\gamma \kappa_q^c q_t, \\ y_t - y_t^* &= \gamma \kappa_q^y q_t, \end{aligned} \quad (2.12)$$

where $\kappa_q^c = \frac{2\theta(1-\alpha)(1-\gamma) + \nu + \frac{\nu+\phi}{1-\phi} \frac{2\gamma}{1-2\gamma}}{1 + \sigma\nu[1 + \frac{1}{1-\phi} \frac{2\gamma}{1-2\gamma}]}$ and $\kappa_q^y = \sigma\nu\kappa_q^c - \frac{\nu+\phi}{1-\phi} \frac{2}{1-2\gamma}$.

Interest Rates and Countries' Constraint: The log-linearization of the home and foreign budget constraint, the demand for home-currency bond, and the demand for foreign-currency bond are shown as:

$$\beta b_{t+1} - b_t = nx_t, \quad (2.13)$$

$$i_t = E_t[\sigma \Delta c_{t+1} + \Delta p_{t+1}],$$

$$\beta b_{t+1}^* - b_t^* = nx_t^*,$$

$$i_t^* = E_t[\sigma \Delta c_{t+1}^* + \Delta p_{t+1}^*]. \quad (2.14)$$

Replacing $c_{t+1}^* - c_{t+1} = \gamma \kappa_q^c q_{t+1}$ and $p_{t+1}^* - p_{t+1} = -\frac{2}{1-\phi} \frac{\gamma}{1-2\gamma} q_{t+1}$, one can express the interest rate differential in terms of the expected exchange rate return:

$$i_t^* - i_t = \gamma \lambda_1 E_t \Delta s_{t+1}, \quad (2.15)$$

where $\lambda_1 = \frac{\sigma \kappa_q^c - \frac{1}{1-\phi} \frac{2}{1-2\gamma}}{1 + \frac{1}{1-\phi} \frac{2\gamma}{1-2\gamma}}$. The home budget constraint relates with the exchange rate as:

$$\beta b_{t+1} - b_t = \gamma \lambda_2 s_t, \quad (2.16)$$

where $\lambda_2 = \frac{\kappa_q^{nx}}{1 + \frac{1}{1-\phi} \frac{2\gamma}{1-2\gamma}}$, $\kappa_q^{nx} = \left[2\theta(1-\alpha) \frac{1-\gamma}{1-2\gamma} + 2(1-\gamma)\alpha - 1 - \gamma \kappa_q^y \right] \frac{1}{1-2\gamma}$, and $\kappa_q^y = \sigma\nu\kappa_q^c - \frac{\nu+\phi}{1-\phi} \frac{2}{1-2\gamma}$. The equation states that the change in net foreign position over time is related to the exchange rate.

UIP Condition: The financial market clearing conditions require $B_{t+1} + N_{t+1} + D_{t+1} = 0$ and $B_{t+1}^* + N_{t+1}^* + D_{t+1}^* = 0$. Given that noise traders and arbitrageurs hold zero-capital position, $N_{t+1} = -S_t N_{t+1}^*$ and $D_{t+1} = -S_t D_{t+1}^*$, the net foreign assets of the home country equal the net liabilities of foreign: $B_{t+1} = -S_t B_{t+1}^*$. Replacing the demand by noise traders, equation (2.8), and the demand by arbitrageurs, equation (2.9), and using $B_{t+1} = -S_t B_{t+1}^*$, I have the following market clearing condition:

$$\frac{B_{t+1}}{S_t} = n(e^{\psi_t} - 1) + \frac{mE_t\tilde{R}_{t+1}}{\omega\text{var}_t\tilde{R}_{t+1}}.$$

From this equation, I can derive for the UIP condition as follows:⁵

$$i_t^* - i_t + E_t\Delta s_{t+1} = \chi_1 b_{t+1} - \chi_2 \psi_t, \quad (2.17)$$

where $\chi_1 = \frac{\bar{P}\bar{Y}}{m/(\omega\sigma_e^2)}$, $\chi_2 = \frac{n/\beta}{m/(\omega\sigma_e^2)}$.

In the model with no financial sector, the deviation from UIP is only driven by the financial shock (ψ_t). In the model with financial sector, the deviation from the UIP not only comes from the financial shock, but also the demand for home currency bonds. This introduces a country's net foreign asset position as one of the determinants of the currency risk premium. Equation (2.17) states that the excess return on foreign deposits depends on two forces. The first one is an exogenous shock on the demand for foreign bonds, ψ_t . The second one is an endogenous force through a state variable in the model known as the net foreign asset position of the home households, b_{t+1} , reflecting the demand for home currency bonds. The excess return on foreign deposits is decreasing with the financial shock, but increasing with a positive net foreign asset position of the home country.

⁵The derivation is shown in Appendix B.2

2.3.2 The Estimation Model

I develop the model for estimation from the finding of equation (2.17) where I can relate the variables of interest, namely, the interest rate differential, the real exchange rate and the net foreign asset, and blend them together in an estimation model. First, I substitute for the interest differential, equation (2.15), in the UIP equation (2.17) to derive:

$$E_t \Delta s_{t+1} = \frac{\chi_1}{1 + \gamma \lambda_1} b_{t+1} - \frac{\chi_2}{1 + \gamma \lambda_1} \psi_t. \quad (2.18)$$

Define the expected excess return on foreign bonds or the risk premium as $E_t \rho_{t+1} = i_t^* - i_t + E_t \Delta s_{t+1}$. Equation (2.16) relates the net foreign asset position and the exchange rate, and (2.11) connects the relationship between the nominal exchange rate and the real exchange. Combining together with (2.18), I can derive $E_t \rho_{t+1}$ as follows:

$$E_t \rho_{t+1} = (i_t^* - i_t) + \frac{\chi_1}{1 + \gamma \lambda_1} \frac{\kappa_q^{nx}}{\beta} q_t + \frac{\chi_1}{1 + \gamma \lambda_1} \frac{1}{\beta} b_t - \frac{\chi_2}{1 + \gamma \lambda_1} \psi_t. \quad (2.19)$$

The currency risk premium thus depends on the interest rate differential, the real exchange rate, the net foreign asset position of the home country, and the demand for the foreign asset shock. Let $\epsilon_{t+1} = \rho_{t+1} - E_t \rho_{t+1}$, equation (2.19) leads to the estimation equation as:

$$\rho_{t+1} = \alpha + \beta_0 (i_t^* - i_t) + \beta_1 q_t + \beta_2 b_t + \epsilon_{t+1}, \quad (2.20)$$

where $\epsilon_{t+1} = -\frac{\chi_2}{1 + \gamma \lambda_1} \psi_t + \epsilon_{t+1}$.

2.4 Empirical Estimation

In this section, I explain about data used in the study. Then, the method of estimation is discussed. Lastly, I show and analyze the empirical results.

2.4.1 Data Analysis

The data used in this study is quarterly and spans in 1980Q1–2016Q3. The sample covers 7 countries (currencies): Australia (AUD), Canada (CAD), Japan (JPY), New Zealand (NZD),

Norway (NOK), Switzerland (CHF), and United Kingdom (GBP). I assume that all these individual countries are home countries and the United States is the foreign country. Exchange rates are end-of-quarter values. All exchange rates are expressed as the home currency per unit of USD. An increase in the exchange rate means that the home currency is depreciating (the foreign currency is appreciating). I obtain data from the International Financial Statistics (IFS), provided by the International Monetary Fund (IMF). This data covers exchange rates, price indexes, interest rates, and gross domestic products. For the nominal interest rate, I use the three months treasury bill interest rates.

2.4.1.1 A Measure of External Imbalances

The log-linearized of demand for home currency bonds in the theoretical model, denoted by b_t , will be captured by a country's net foreign asset position (NFA). The NFA position of a country measures the difference in the value of foreign assets that the country owns and the value of domestic assets owned by foreigners. If a country has positive (negative) NFA, it is a creditor (debtor) to the rest of the world. NFA can be calculated as the sum of foreign assets held by a country less its foreign liabilities. As in Lane and Milesi-Ferretti (2001, 2007), NFA is computed by:

$$\begin{aligned} NFA = & \text{(Foreign Direct Investment Assets-Foreign Direct Investment Liabilities)} \\ & + \text{(Portfolio Equity and Debt Assets-Portfolio Equity and Debt Liabilities)} \\ & + \text{(Foreign Reserves)}. \end{aligned}$$

The accumulation identity for NFA between periods t and $t + 1$ is:

$$NFA_{t+1} \equiv R_{t+1}(NFA_t + NX_t),$$

where NFA_t is the difference between gross external assets A_t and gross external liabilities L_t ; NX_t is net exports which is defined as exports X_t minus imports M_t of goods and services; and R_{t+1} is the return on the NFA portfolio. A country's NFA improves with positive net exports. Also, part of the adjustment of the external imbalances occurs through the return on the NFA portfolio.

The actual data of NFA/GDP for each country, shown in Figure 2.1–2.8, has either upward or downward trends. To exploit this data in the estimation, we need to remove the trend component, and focus on its stationary part. Gourinchas and Rey (2007) study the log-linearization of the external constraint and separate the data into the trend component and the stationary component. The approximation of the external constraint around its trend satisfies

$$nfa_{t+1} \approx \frac{1}{\rho} nfa_t + r_{t+1} + \Delta nx_{t+1}, \quad (2.21)$$

where

$$\begin{aligned} nfa_t &\equiv |\mu^a| \epsilon_t^a - |\mu^l| \epsilon_t^l + |\mu^x| \epsilon_t^x - |\mu^m| \epsilon_t^m, \\ \Delta nx_{t+1} &\equiv |\mu^x| \Delta \epsilon_{t+1}^x - |\mu^m| \Delta \epsilon_{t+1}^m - \epsilon_{t+1}^{\Delta w}, \\ r_{t+1} &\equiv \frac{\mu^a}{|\mu^a|} \hat{r}_{t+1}, \\ \rho &\equiv 1 + \frac{\bar{X} - \bar{M}}{\bar{A} - \bar{L}}. \end{aligned} \quad (2.22)$$

Define $Z_t \in \{A_t, L_t, X_t, M_t\}$. The stationary component is defined as $\epsilon_t^z \equiv \ln(\hat{Z}_t/\bar{Z}_t)$, where $\hat{Z}_t = Z_t/W_t$, \bar{Z}_t is the equilibrium value of Z_t/W_t , and W_t is a wealth at time t . The term nfa_t in equation (2.22) is a measure of cyclical external imbalances. It is a linear combination of the stationary components of (log) assets, liabilities, exports, and imports to wealth ratios. They are denoted by ϵ_t^a , ϵ_t^l , ϵ_t^x , and ϵ_t^m , respectively. These stationary components are multiplied by the weight μ^z . The term μ^a denotes the share of assets in the net foreign assets, and is calculated by $\frac{\bar{A}}{\bar{A}-\bar{L}}$, while $\mu^l = \mu^a - 1$. The term μ^x represents the (trend) share of exports in the trade balance, $\mu^x = \frac{\bar{X}}{\bar{X}-\bar{M}}$, and $\mu^m = \mu^x - 1$.

I will use nfa defined above as a proxy for a country's external imbalances. The data used in this calculation is taken from IMF's Balance of Payments (BOP) Statistics, which reports the data by BOP components and the international investment position (IIP). To construct variables in the stationary components, I apply the Hodrick-Prescott (HP) filter to remove trend component out of the data. For example, to construct ϵ_t^a , I first calculate $\ln(A_t/GDP_t)$, and then I apply the HP filter to this ratio. The cycle component from the HP filter is the stationary component of the log of the ratio of assets to wealth.

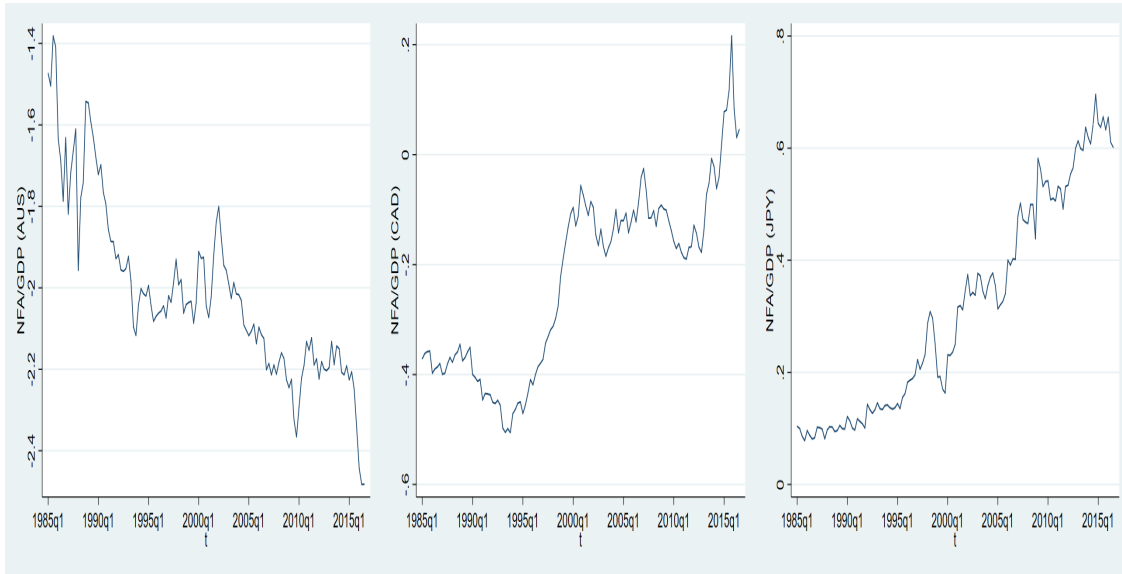


Figure 2.1: Australia

Figure 2.2: Canada

Figure 2.3: Japan

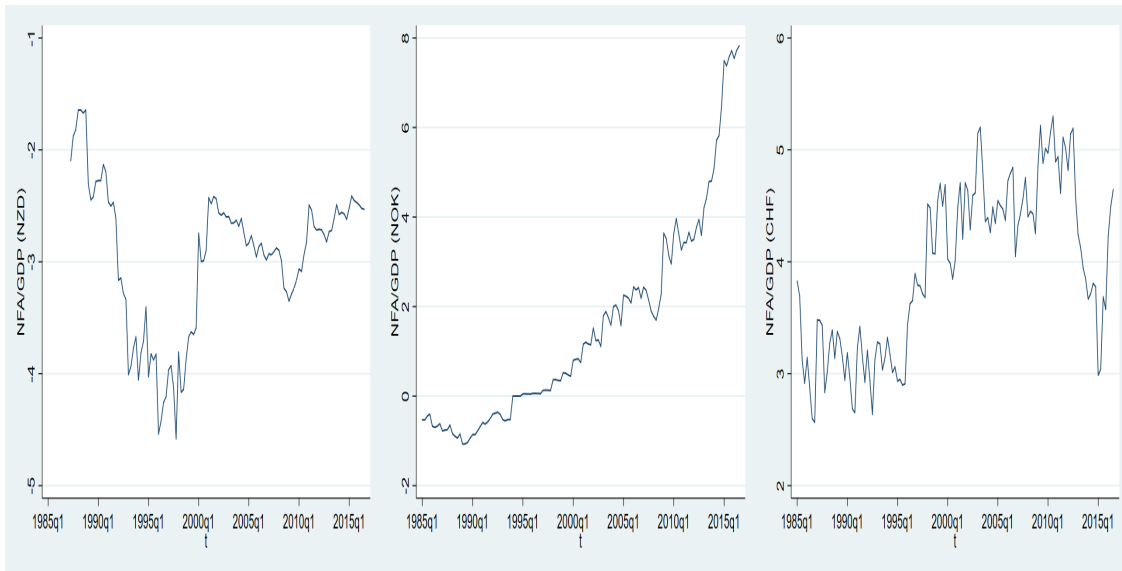


Figure 2.4: New Zealand

Figure 2.5: Norway

Figure 2.6: Switzerland

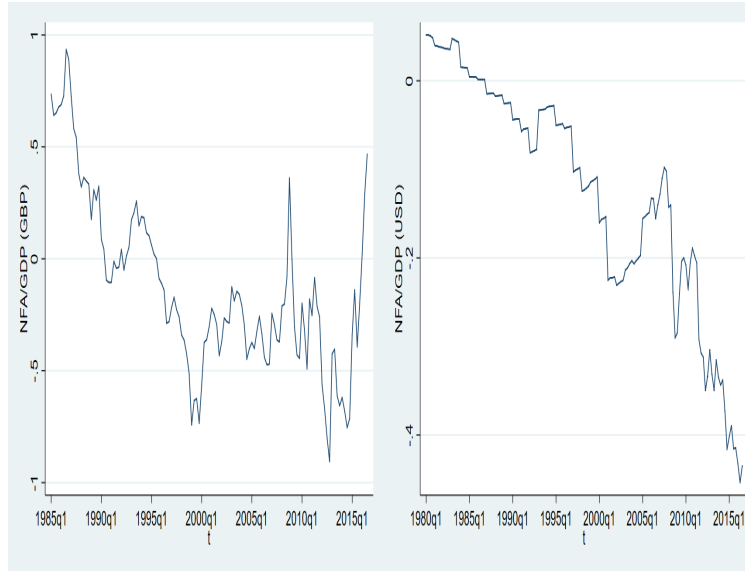


Figure 2.7: United Kingdom Figure 2.8: Unites States

2.4.1.2 Summary Statistics

The summary statistics are reported in Table 2.1. In this table, means, standard deviations, minimum and maximum values of currency excess returns, interest rate differentials, real exchange rates, and external imbalances of each country in the sample are reported. The average of the currency excess return in the sample is quite low and negative. Most of currencies give negative excess return except for the excess return between JPY/USD and CHF/USD. The standard deviation of the currency excess return ranges between 0.0390 and 0.0635. Return volatilities are quite high. The interest rate differentials ($i_t^* - i_t$) are negative, suggesting that each country has a higher interest rate than the US. However, the interest rates in Japan and Switzerland are lower than in the US.

2.4.2 Estimation Method

Since the data observed are the same cross section units (countries) at different points in time, I apply this data set with panel estimation method. In panel data analysis, an unobserved, time-

Table 2.1: Summary Statistics

Variable	Statistic	AUS	CAD	JPY	NZD	NOK	CHF	GBP	ALL
ρ_{t+1}	Mean	-0.0189	-0.0114	0.0221	-0.0373	-0.0084	0.0049	-0.0214	-0.0098
	Std.Dev.	0.0605	0.0390	0.0619	0.0598	0.0601	0.0635	0.0549	0.0601
	Min	-0.1808	-0.0987	-0.1292	-0.2465	-0.1811	-0.1644	-0.1724	-0.2465
	Max	0.1607	0.1383	0.1927	0.1312	0.1752	0.1361	0.1763	0.1927
$i_t^* - i_t$	Mean	-0.0195	-0.0112	0.0290	-0.0354	-0.0074	0.0123	-0.0204	-0.0072
	Std.Dev.	0.0148	0.0156	0.0113	0.0233	0.0180	0.0193	0.0202	0.0267
	Min	-0.0674	-0.0583	0.0061	-0.1557	-0.0646	-0.0521	-0.0714	-0.1557
	Max	-0.0004	0.0210	0.0503	0.0021	0.0167	0.0399	0.0050	0.0503
q_t	Mean	0.3254	0.1749	4.5150	0.4581	1.8946	0.1350	-0.5098	1.0046
	Std.Dev.	0.1901	0.1262	0.1667	0.1803	0.1323	0.1455	0.0943	1.6014
	Min	-0.0693	-0.0331	4.0658	0.1442	1.6556	-0.1569	-0.7324	-0.7324
	Max	0.7623	0.4353	4.9414	0.9401	2.2171	0.6486	-0.2093	4.9414
nfa_t	Mean	-0.0080	-0.0511	0.0317	$1.28e - 9$	-0.0020	0.0013	0.0294	0.0002
	Std.Dev.	0.9006	1.0742	0.7657	1.0138	0.3190	0.2750	1.1914	0.8556
	Min	-1.7359	-2.3765	-1.9458	-2.9096	-1.0042	-1.2105	-3.1201	-3.1201
	Max	2.0535	2.6767	2.2806	2.5085	1.0432	0.6887	3.2624	3.2624

constant variable is called an unobserved effect. This unobserved effect captures features of an individual country characteristics that are given and do not change over time. I adopt the fixed effect framework as it allows for arbitrary dependence between the unobserved effect and the observed explanatory variables. In this way, the fixed effect analysis is more robust than random effect analysis.

Bilateral External Imbalances and Instrumental Variables: The definition of nfa mentioned before is actually called the global nfa since it measures a country's net foreign assets and liabilities against the rest of the world. However, what is actually desired in this study is an nfa between two countries of interest (home and foreign), called a bilateral nfa , because the study is framed on a two-country basis. Unfortunately, a bilateral measure of nfa is not directly observable because data on a bilateral basis are not available. If global nfa is used in a regression for currency excess returns between two countries, it would cause a measurement error, leading to inconsistent

least squares estimates.⁶ I apply the instrument variable (IV) technique.⁷ The IV requires a set of instruments that contain information of bilateral *nfa* that is correlated with (home) global *nfa* but uncorrelated with the measurement error.

Della Corte et al. (2012) study the ability of external imbalances to predict exchange rates. In the context of their paper, the bilateral *nfa* is used to predict bilateral exchange rates. Since they can only obtain data on global external imbalances, not bilateral ones, they propose two instruments. The first one is the global *nfa* for foreign country. In my study, it refers to the global *nfa* of the US. The global *nfa* of the US is composed of two parts: the bilateral *nfa* between US and home country, and the *nfa* between US and the rest of the world. Thus the global *nfa* of the US contains information of the bilateral *nfa* between home and US, and also correlates with home country global *nfa*. The second instrument is the bilateral detrended net exports between home country and the US, denoted by nx_t^i . It is constructed as a linear combination of the stationary components of (log) bilateral exports and imports to GDP ratios. According to the identity equation of the NFA and (2.22), the bilateral net exports should relate to the bilateral *nfa*.

2.4.3 Estimation Results

To investigate the influence of interest rates, real exchange rates, and external imbalances, I study 4 models. The first model is the Fama regression, in which the currency risk premium is described by the interest rate differential. The second model is augmented from the first one by adding the real exchange rate as an explanatory variable. The third model regresses the currency risk premia on interest rate differential and the external imbalance, measured by *nfa*. The last regression includes all three explanatory variables: the interest rate differential, the real exchange rate, and the external imbalance. Model 4 is a model that I propose in this study. The four models are summarized below.

⁶For example, if the true model is $y = x\beta + u$, $E(u|x) = 0$ implying that $cov(x, u) = 0$. x is unobserved, instead we observed $x^* = x + \varepsilon$. This is called measurement error. If we run an OLS regression of $y = x^*\beta + v$ where $v = (u - \beta\varepsilon)$, we will have endogeneity problem because $cov(x^*, v) \neq 0$.

⁷Suppose IV is $x^{**} = x + \eta$ where η is some error. I apply the estimation method using IV, called two-stage least square (2SLS). In the first stage regression, I regress $x^* = x^{**}\gamma + \delta$ which could predict the estimate of x^* as \hat{x}^* . In the second stage, I regress $y = \hat{x}^*\beta + v$ which would yield $\hat{\beta}_{2SLS}$ that is a consistent estimate of β .

$$\text{Model 1: } \rho_{j,t+1} = \beta_0 + \beta_1(i_t^* - i_{j,t}) + \epsilon_{j,t+1}.$$

$$\text{Model 2: } \rho_{j,t+1} = \beta_0 + \beta_1(i_t^* - i_{j,t}) + \beta_2q_{j,t} + \epsilon_{j,t+1}.$$

$$\text{Model 3: } \rho_{j,t+1} = \beta_0 + \beta_1(i_t^* - i_{j,t}) + \beta_2nfa_{j,t} + \epsilon_{j,t+1}.$$

$$\text{Model 4: } \rho_{j,t+1} = \beta_0 + \beta_1(i_t^* - i_{j,t}) + \beta_2q_{j,t} + \beta_3nfa_{j,t} + \epsilon_{j,t+1}.$$

The results of panel estimation of the four models are reported in Table 2.2. In this study, I attempt to address the importance of the panel analysis that allows for differences in country characteristics. Most of the previous literature consists of studies using a time series framework. The results from the Fama regression, or Model 1, are reported in the second column. As usually found in the literature, the estimates of β_1 are positive and statistically significant. This is also true for models 2, 3, and 4, for which the coefficients of $i_t^* - i_t$ are significantly positive. The result implies that high interest rate currencies earn high returns. This finding also highlights the well-known UIP puzzle that a higher interest rate currency tends to appreciate in value.

Dahlquist and Penasse (2016) argue that the Fama regression may be affected by an omitted variable bias and suggest to include the real exchange rate in the regression, as appearing in Model 2. When the regression incorporates the real exchange rate, q_t , I find that the coefficient of q_t is negatively significant. When the real exchange rate is high (the foreign currency is expensive relative to the home currency), the excess return on foreign deposits tend to be lower. This finding is consistent with Dahlquist and Penasse (2016) and Balduzzi and Chiang (2017) that real exchange rate does predict currency returns. Furthermore, adding the real exchange rate in the regression increases the adjusted R^2 from 14.58% in the Fama model to 17.22%, an 18% increase.

In model 3, the Fama regression is augmented by nfa_t . While the coefficient of $i_t^* - i_t$ is positive and statistically significant, the coefficient of nfa_t has no statistical significance. The adjusted R^2 values worsen in comparison to models 1 and 2. Thus, the model of currency risk premia with interest rate differential and external imbalance does not seem to work well. It is perhaps not the best model.

In model 4, all three variables of interest are included in the regression. This estimation model is directly derived from the theoretical framework. Similarly to previous results, coefficient of $i_t^* - i_t$ is positive and statistically significant, and the coefficient of q_t is significantly negative. The coefficient of nfa_t is positive and statistically significant. It implies that currency risk premium increases when the funding currency (home currency) is a net foreign creditor (investment currency is a net debtor). The result supports the finding of Della Corte et al. (2016) that net debtor country offers a currency risk premium to compensate investors who finance negative external imbalances. The Hausman test for endogeneity rejects the null hypothesis of exogeneity. I then apply the IV method to correct for the endogeneity problem. The validity of the two instrument variables, the global nfa of foreign country and the bilateral net exports, is tested by the Hansen test. The Hansen J statistic suggests that both IVs are valid.

Table 2.2: Estimation Results

Variable	Model 1	Model 2	Model 3	Model 4
Constant	0.0007**	0.0665**	-0.0024***	0.0938***
(s.e)	(0.0002)	(0.0106)	(0.0004)	(0.0223)
$i_t^* - i_t$	1.2000***	1.3266***	1.0318***	1.1004***
(s.e)	(0.0376)	(0.0716)	(0.0513)	(0.0578)
q_t		-0.0635**		-0.0953***
(s.e)		(0.0100)		(0.0220)
nfa_t			0.0129	0.0283***
(s.e)			(0.0090)	(0.0068)
Adj R^2	0.1458	0.1722	0.0486	0.0317
Hausman Test			4.8759*	8.7802**
p-val			[0.0873]	[0.0324]
Hansen Statistic			0.003	1.996
p-val			[0.9596]	[0.1577]

*, **, *** denote significance at level 10%, 5%, 1% respectively.

Cochrane (2011) finds that interest rate differentials mostly capture information about short-term currency returns. I also investigate the long-horizon prediction of the currency excess returns to study whether the real exchange rates and the external imbalances capture long-term currency returns. Table 2.3 displays the results for cumulative excess return predictability over long-horizons.

For example, the cumulative excess returns is $\rho_{t,t+j} = \sum_{k=1}^j \rho_{t+k}$. The R^2 values are increasing

with $t + k$ horizon in every model. Moreover, the R^2 of the long-horizon prediction increases the most in model 4, from about 3% in the one-quarter horizon to about 47% in the eight-quarter horizon. This implies that the real exchange rates, and the external imbalances strengthen the excess return predictability over the investment horizon, and hence they can capture information on long-term currency returns.

Table 2.3: Long Horizon Prediction

Model	Variable	$\rho_{t,t+1}$	$\rho_{t,t+2}$	$\rho_{t,t+4}$	$\rho_{t,t+8}$
Model 1	$i_t^* - i_t$	1.2000***	2.2500***	3.9841***	6.0196***
	(s.e)	(0.0376)	(0.1085)	(0.3042)	(0.7057)
	R^2	0.1458	0.2203	0.2893	0.2655
Model 2	$i_t^* - i_t$	1.3266***	2.5299***	4.5669***	7.2713***
	(s.e)	(0.0716)	(0.1692)	(0.4082)	(0.9303)
	q_t	-0.0635**	-0.1401***	-0.2919***	-0.6212***
	(s.e)	(0.0100)	(0.0217)	(0.0421)	(0.0793)
Model 3	$i_t^* - i_t$	1.0318***	2.0636***	4.1145***	6.7405***
	(s.e)	(0.0513)	(0.1117)	(0.3092)	(0.7021)
	nfa_t	0.0129	-0.0029	-0.0579	-0.1273
	(s.e)	(0.0090)	(0.0209)	(0.0411)	(0.0685)
Model 4	$i_t^* - i_t$	1.1004***	2.1790***	4.2745***	7.0539***
	(s.e)	(0.0578)	(0.0905)	(0.2521)	(0.6193)
	q_t	-0.0953***	-0.1867***	-0.3321***	-0.6722***
	(s.e)	(0.0220)	(0.0446)	(0.0655)	(0.1164)
Model 4	nfa_t	0.0283***	0.0313*	0.0109	0.0117
	(s.e)	(0.0068)	(0.0179)	(0.0232)	(0.0552)
	R^2	0.0317	0.0983	0.3738	0.4736

*, **, *** denote significance at level 10%, 5%, 1% respectively.

2.4.4 Forecasting

So far the results show that, apart from interest rate differentials, real exchange rates and external imbalances are significant factors in determining currency excess returns. Now, I try to answer which of the four models is the best in forecasting currency excess returns. In this section, I will gauge the best model by its ability to forecast the currency excess returns out-of-sample. First, I keep the data from 1980Q1–2001Q4 as sample periods. I run regressions for each model

and predict currency returns recursively over an expanding window from 2002Q1 onwards. Then, I evaluate the forecast by comparing the mean squared prediction error (MSPE) for each model. The model with lower MSPE is best.

Furthermore, I assess the statistical significance of the MSPE using the Clark and West (2007) method. The idea is to use the point estimate of the difference between the MSPEs of the two models, adjusting for the noise associated with the nested model's forecast. Forecast evaluation compares the MSPE of the parsimonious model (or the benchmark model) with a larger alternative that nests the parsimonious model. Under the null hypothesis that additional parameters in the alternative model are not useful for prediction, the MSPE of the parsimonious model should be smaller than that of the alternative model.

Let model 0 be the parsimonious model and model a be the larger model that nests model 0. Model a is nested in model 0 if model a reduces to model 0 when some parameters in model a are set to zero. I am interested in forecasting excess returns 1-step ahead. The period t forecasts of y_{t+1} from the two models are $\hat{y}_{t,t+1}^0$ and $\hat{y}_{t,t+1}^a$. The forecasting errors are $y_{t+1} - \hat{y}_{t,t+1}^0$ and $y_{t+1} - \hat{y}_{t,t+1}^a$. The statistic for difference MSPE-adjusted defines as:

$$\hat{f}_{t+1} = (y_{t+1} - \hat{y}_{t,t+1}^0)^2 - [(y_{t+1} - \hat{y}_{t,t+1}^a)^2 - (\hat{y}_{t,t+1}^0 - \hat{y}_{t,t+1}^a)^2].$$

The null hypothesis is that the MSPE of the benchmark model is lower than or equal to the MSPE of model a . The alternative is that model a has smaller MSPE. I test this hypothesis using t-test on the average of \hat{f}_{t+1} .

Many studies of stock return predictability show that predictive regressions of stock returns performed poorly out-of-sample. Welch and Goyal (2008) find that the historical average excess stock return forecasts future excess returns better than regressions of excess returns on predictor variables, i.e. dividend-price ratio, earning-price ratio, and book-to-market ratio. I test the ability of the historical average against models 1–4 in forecasting currency excess returns. Table 2.4 reports the t-statistics and p-values in testing significance of difference MSPE-adjusted. The null hypothesis is that the historical average model has lower or equal MSPE to each individual model 1–4. There is statistical significance that models 1, 2, 3, and 4 beat the historical average currency returns in

Norway, Switzerland, and United Kingdom. Only models 2 and 4 perform better than the historical average model in Canada. The results show that models 1–4 cannot beat the historical average for Australia, Japan, and New Zealand. The equally weighted portfolio of the seven currencies is referred to as “Portfolio”. As seen, judging from the portfolio, models 2 and 4 perform better in forecasting currency returns than the historical average. Adding the real exchange rates to interest rate differentials, and adding both real exchange rates and external imbalances to interest rate differentials would help forecasting currency excess returns out-of-sample better than using the historical average of currency excess returns to forecast its future.

Table 2.4: Testing Historical Averages Against Models 1–4

Historical Average against	AUD	CAD	JPY	NZD	NOK	CHF	GBP	Portfolio
Model 1	-0.2666	0.7010	0.0139	-1.1501	2.2774**	1.5838*	2.9711**	0.3279
p-value	0.6051	0.2416	0.4945	0.8749	0.0114	0.0566	0.0015	0.3715
Model 2	1.0554	1.5924*	0.7175	0.7560	2.7742**	2.1209**	3.7549***	1.8580**
p-value	0.1456	0.0556	0.2365	0.2248	0.0028	0.0170	0.0000	0.0326
Model 3	-1.5295	0.1650	-0.4537	0.0007	2.3074**	1.6722**	2.2629**	-0.0669
p-value	0.9369	0.4345	0.6750	0.4997	0.0105	0.0472	0.0118	0.5267
Model 4	0.0976	2.5518**	-0.1338	0.2498	3.0794**	1.9476**	2.5554**	1.8574**
p-value	0.4611	0.0054	0.5532	0.4014	0.0010	0.0257	0.0053	0.0316

*, **, *** denote significance at level 10%, 5%, 1% respectively.

Since models 2, 3, and 4 are nested in model 1, I also test for the ability of the out-of-sample currency returns forecast of model 1 against these nested models. This is to determine whether adding variables, e.g. real exchange rates, and external imbalances, together with interest rate differentials help forecast currency excess returns better than using interest rate differentials alone. The results are shown in Table 2.5. Adding real exchange rates into the model of currency returns with interest rate differentials increases the forecasting ability of the portfolio at the 10% level of significance. But adding external imbalances alone is not statistically significantly better than model 1. For each individual country, only Switzerland exhibits statistical significance that model 2 performs better than model 1. That is, only considering the whole portfolio and Switzerland individually, the model addition with real exchange rates performs better in forecasting currency

returns using interest rate differentials alone. Model 3 is not significantly better than model 1 for all countries, including the portfolio. These results suggest that adding external imbalances does not help improve the ability to forecast currency excess return. However, model 4 beats model 1 for 3 countries: Canada, Norway, and Switzerland, including the portfolio of seven currencies. This implies that to increase the ability to forecast future currency excess returns, it is necessary to add both real exchange rates and external imbalances to interest rate differentials.

Table 2.5: Testing Model 1 Against Models 2–4

Model 1 against	AUD	CAD	JPY	NZD	NOK	CHF	GBP	Portfolio
Model 2	1.0704	0.8852	0.7091	1.2640	1.0649	1.6466**	0.7850	1.4570*
p-value	0.1422	0.1880	0.2391	0.1031	0.1435	0.0498	0.2161	0.0726
Model 3	-0.9151	0.6802	-1.205	1.2017	0.9890	0.5023	0.4162	0.2689
p-value	0.8199	0.2482	0.8859	0.1147	0.1613	0.3077	0.3386	0.3940
Model 4	0.0114	2.4763**	-1.0086	0.0136	1.9517**	1.7875**	0.7561	1.5367*
p-value	0.4954	0.0066	0.8434	0.4946	0.0255	0.0369	0.2248	0.0622

*, **, *** denote significance at level 10%, 5%, 1% respectively.

With interest rate differentials, does adding both real exchange rates and external imbalances increase excess return predictability over the models that only add either real exchange rates or external imbalances? To answer this question, I perform a test by adjusting the MSPE of model 2 against model 4, and model 3 against model 4. Tables 2.6 and 2.7 report these results. I find that the null hypothesis is rejected only for Canada and Norway. That is, model 4 is better than model 2. So it is the case for Canada and Norway that the model adding both real exchange rates and external imbalances to interest rate differentials outperforms the model with interest rate differentials adding real exchange rates alone. For other countries that do not reject the null, these results imply that the model adding both variables does not perform better than the model adding real exchange rates alone in forecasting currency returns. In comparison with model 3 and 4, I find that adding both variables to interest rate differentials outperforms the model with interest rate differentials adding external imbalances alone for Canada, Norway, Switzerland, and the portfolio

of seven currencies. Model 4, adding both variables, does not perform better in forecasting currency returns than model 3 (with external imbalances) for the rest of the countries.

In conclusion, model 4 outperforms model 1, suggesting that, apart from interest rate differentials, we should add both real exchange rates and external imbalances in predicting currency excess returns. Models 2 and 3 are not statistically better than model 1. However, there is no strong evidence that adding both real exchange rates and external imbalances to the model of currency excess returns with interest rate differentials would perform better than the one with interest rate differentials and real exchange rates. Model 4 is better than model 2 for only two out of seven countries. It is also true for three countries and the overall portfolio that the model adding for both real exchange rates and external imbalances would be better than the model of currency excess returns with interest rate differentials and external imbalances. Therefore, adding only one more factor to interest rate differentials does not help improve forecasting. To forecast currency excess returns, it would be best to have all three variables: interest rate differentials, real exchange rates, and external imbalances together with.

Table 2.6: Testing Model 2 Against Model 4

Model 2 against	AUD	CAD	JPY	NZD	NOK	CHF	GBP	Portfolio
Model 4	-1.3642	2.2364**	-2.6910	-2.1275	2.1753**	0.9830	0.6653	0.8126
p-value	0.9137	0.0127	0.9964	0.9833	0.0148	0.1628	0.2529	0.2082

*, **, *** denote significance at level 10%, 5%, 1% respectively.

Table 2.7: Testing Model 3 Against Model 4

Model 3 against	AUD	CAD	JPY	NZD	NOK	CHF	GBP	Portfolio
Model 4	0.3381	1.9094**	-0.9598	-0.5921	1.7839**	1.6899**	0.3458	1.6753**
p-value	0.3677	0.0281	0.8314	0.7231	0.0372	0.0455	0.3647	0.0469

*, **, *** denote significance at level 10%, 5%, 1% respectively.

2.5 Conclusion

The violation of UIP suggests that interest rate differentials have a positive relationship with currency excess returns. The high interest rate currency offers high excess returns. In this chapter, I study the role of economic variables other than interest rate differentials on currency excess returns. I focus on the role of real exchange rates and country's external imbalances. I draw attention to the importance of these two variables by means of a dynamic general equilibrium model of exchange rate determination proposed by Itskhoki and Mukhin (2017). I then develop estimation models and assess their ability to forecast future currency excess returns.

I find that real exchange rates and external imbalances are important factors in explaining variation in currency excess returns. Real exchange rates are negative related to currency excess returns. When real exchange rates are high, excess returns on foreign deposits are low because the foreign currency is expensive compared to the home currency. Net debtor countries provide high currency risk premia because investors need to be compensated to be willing to finance negative external imbalances. In terms of future excess returns forecasting, I find that the model that incorporates both real exchange rates and external imbalances to interest rate differentials outperforms the model that forecasting currency excess returns using interest rate differentials alone. Adding only real exchange rates or only external imbalances does not help forecasting ability that much. It would be best for a model predicting currency excess returns to involve interest rate differentials, real exchange rates, and external imbalances. Not one or two factors, but all these three variables together with would produce the best model for currency excess returns determination.

2.6 References

- Backus, David K.; Foresi, Silverio and Telmer, Chris I. "Affine Term Structure Models and the Forward Premium Anomaly." *Journal of Finance*, 2001, 56(1), pp. 279-304.
- Balduzzi, Poerluigi and Chiang, I-Hsuan Ethan. "Real Exchange Rate and Currency Risk Premia." Working Paper, 2017.

- Bansal, Ravi and Shaliastovich, Ivan. "A Long-Run Risks Explanation of Predictability Puzzles in Bond and Currency Markets." *Review of Financial Studies*, 2013, 26(1), pp. 1-33.
- Brunnermeier, Markus K.; Nagel, Stefan and Pedersen, Lasse H. "Carry Trade and Currency Crashes." *NBER Macroeconomics Annual*, 2008, pp. 313-347.
- Burnside, Craig; Eichenbuam, Martin; Kleshchelski; Issac and Rebelo, Sergio. "The Returns to Currency Speculation." NBER Working Paper, 2006.
- Burnside, Craig; Eichenbuam, Martin and Rebelo, Sergio. "Carry Trade: The Gains of Diversification." *Journal of the European Economic Association*, 2008, 6, pp. 581-588.
- Burnside, Craig; Eichenbuam, Martin and Rebelo, Sergio. "Carry Trade and Momentum in Currency Markets." *Annual Review of Financial Economics*, 2011, 3, pp. 511-535.
- Clark, Todd E. and West, Kenneth D. "Approximately Normal Tests for Equal Predictive Accuracy in Nested Models." *Journal of Econometrics*, 2007, 138, pp. 291-311.
- Cochrane, John H. "Presidential Address: Discount Rates." *Journal of Finance*, 2011, 66, pp. 1047-1108.
- Colacito, Riccardo and Croce, Mariano M. "International Asset Price Pricing with Recursive Preferences." *Journal of Finance*, 2013, 68(6), pp. 2651-2686.
- Dahlquist, Magnus and Penasse, Julien. "The Missing Risk Premium in Exchange Rates." Working Paper, 2016.
- Della Corte, Pasquale; Sarno, Lucio and Sestieri, Giulia. "The Predictive Information Content of External Imbalances for Exchange Rate Returns: How Much Is It Worth?" *The Review of Economics and Statistics*, 2012, 94(1), pp. 100-115.
- Della Corte, Pasquale; Riddiough J., Steven and Sarno, Lucio. "Currency Premia and Global Imbalances." *The Review of Financial Studies*, 2016, 29(8), pp. 2161-2193.
- Engel, Charles. "The Forward Discount Anomaly and the Risk Premium: A Survey of Recent Evidence." *Journal of Empirical Finance*, 1996, 3, pp. 123-192.
- Engel, Charles. "Exchange Rates, Interest Rates, and the Risk Premium." *American Economic Review*, 2016, 106, pp. 436-474.
- Engel, Charles and West, Kenneth D. "Exchange Rates and Fundamentals." *Journal of Political Economy*, 2005, 113, pp. 485-517.
- Fama, Eugene F. "Forward and Spot Exchange Rates." *Journal of Monetary Economics*, 1984, 14, pp.319-338.
- Froot, Kenneth A. and Thaler, Richard H. "Anomalies: Foreign Exchange." *Journal of Economic Perspectives*, 2001, 4, pp. 179-192.
- Gabaix, Xavier and Maggiori, Matteo. "International Liquidity and Exchange Rate Dynamics." *Quarterly Journal of Economics*, 2015, 130, pp. 1369-1420.

- Gourinchas, Pierre-Oliver and Rey, Hélène. "International Financial Adjustment." *Journal of Political Economy*, 2007, 115(4), pp. 665-703.
- Itskhoki, Oleg and Mukhin, Dmitry. "Exchange Rate Disconnect In General Equilibrium." Working Paper, 2017.
- Lane, Philip R. and Milesi-Ferretti, Gian Maria. "The External Wealth of Nations: Measures of Foreign Assets and Liabilities for Industrial and Developing Countries." *Journal of International Economics*, 2001, 55, pp. 263-294.
- Lane, Philip R. and Milesi-Ferretti, Gian Maria. "The External Wealth of Nations Mark II: Revised and Extended Estimates of Foreign Assets and Liabilities, 1970-2004." *Journal of International Economics*, 2007, 73, pp. 223-250.
- Lustig, Hanno and Verdelhan, Adrien. "The Cross Section of Foreign Currency Risk Premia and Consumption Growth Risk." *American Economic Review*, 2007, 97, pp. 89-117.
- Lustig, Hanno; Roussanov, Nikolai and Verdelhan, Adrien. "Common Risk Factors in Currency Markets." *Review of Financial Studies*, 2011, 24, pp. 3731-3777.
- Menkhoff, Lukas; Sarno, Lucio; Schmeling, Maik and Schrimpf, Andreas. "Carry Trades and Global Foreign Exchange Volatility." *Journal of Finance*, 2012, 67(2), pp. 681-718
- Menkhoff, Lukas; Sarno, Lucio; Schmeling, Maik and Schrimpf, Andreas. "Currency Value." *Review of Financial Studies*, 2017, 30, pp. 416-441.
- Obstfeld, Maurice and Rogoff, Kenneth. "The Six Major Puzzles in International Macroeconomics: Is There a Common Cause?" *NBER Macroeconomics Annual 2000*, 2001, 15, pp. 339-412.
- Valchev, Rosen. "Exchange Rates and UIP Violations at Short and Long Horizons." Working Paper, 2015.
- Verdelhan, Adrien. "A Habit-Based Explanation of the Exchange Rate Risk Premium." *Journal of Finance*, 2010, 65(1), pp. 29-42.
- Welch, Ivo and Goyal, Amit. "Predicting the Equity Premium with Dividend Ratios." *The Review of Financial Studies*, 2008, 21(4), pp. 1455-1508.

CHAPTER 3. EXCHANGE RATE AND UNCOVERED INTEREST RATE PARITY PUZZLE IN A LONG-RUN RISK MODEL

This study employs the idea that consumption growth prospects contain a long-run risk component with stochastic volatility in order to explain the exchange rate and deviations from uncovered interest rate parity (UIP). The model relates exchange rate, interest rate, and risk premium with a stochastic discount factor. I show that theoretically the stochastic volatility of both short-run and long-run shocks account for a negative UIP coefficient, which suggests the UIP violation. I incorporate monetary policy into the model by specifying the interest rate to contain long-run risk through the Taylor rule. Then I study the effect of both level and volatility shocks, on exchange rate and currency risk premium. I find that the specification of the interest rate rule matters for a magnitude change of the variables of interest in response to shocks but it does not matter for a directional change of these variables. The impact of changes in level shocks and volatility shocks on the exchange rate and the risk premium are different. A decrease in short- and long-run consumption growth level shocks, and an increase in short- and long-run consumption growth volatility shocks lead to an appreciation of the exchange rate. An increase in consumption volatility shock would also raise the excess return on foreign deposits. In terms of monetary policy, an increase in the policy shock leads to an appreciation of the exchange rate and a deviation from UIP in the form of positive excess returns on foreign currency.

3.1 Introduction

Uncovered interest parity (UIP) relates short-term interest rates and currency depreciation rates in the sense that, when investors are risk neutral, any cross-country differences in interest rates are associated with the expected exchange rate depreciation. The standard asset-pricing model usually assumes that UIP holds, and an investor is indifferent to return on deposits between two countries.

However, many literature find that the model is unable to demonstrate the UIP condition, as the exchange rates of high interest rate countries do not depreciate enough to offset interest rate differentials. The seminal work by Fama (1984) evidences the violation of UIP. He actually suggests that currency appreciates in response to a rise in interest rate. The UIP puzzle has led researchers, for example, Backus et al. (2001), Alvarez et al. (2009), Verdelhan (2010), Colacito and Croce (2011, 2013), Bansal and Shaliastovich (2013), to model this deviation through a risk premium that causes a wedge between interest rate differentials and the expected depreciation of exchange rates. This risk premium is paid for a time-varying risk in the sense that high interest rate currencies pay positive premiums.

The deviation from UIP can be accounted for in terms of compensation for risk. Investors require a premium in order to hold a risky asset. So there will be an excess return for holding this risky asset relatively to other assets. To be more concrete, a domestic investor chooses to invest in bonds denominated in home or foreign currency. For this investor, the return on the foreign bond in terms of domestic currency is risky because exchange rate in the next period is not known today. The currency risk premium compensates the investor who holds the foreign bond for taking on the exchange rate risk. It is interesting to examine how variation in risk over time affects exchange rate movements and hence currency risk premium. Alvarez et al. (2009) find that risk premium varies over time because the degree of asset market segmentation varies over time in response to stochastic shocks. Benigno et al. (2001) examines the role of nominal and real stochastic volatilities (risks) for the behavior of exchange rate. They find that volatility shocks are essential for the equilibrium of exchange rate and interest rate. First, they empirically show that an increase in nominal volatility, e.g. volatility of monetary policy shocks and volatility of inflation target shocks, would induce the exchange rate to appreciate. The intuition is that if a currency is a good hedge for a particular risk, the demand for this currency rises in response to an increase in this risk. Then its exchange rate appreciates. However, an increase in real risk, e.g. productivity shock volatility, would induce an exchange rate depreciation. From the UIP perspective, an increase in nominal volatility leads to an increase in the excess return of foreign short-term bonds. Then they develop a New Keynesian

model that allows for a general specification of preferences as in Epstein and Zin (1989); nominal price rigidities as in Calvo's (1983) model, and stochastic volatility in the exogenous processes of the economy to support their findings. The important part is the specification of monetary policy through interest rate rules and the presence of a stochastic volatility term that play a vital role to create a negative coefficient in an UIP regression.

Bansal and Yaron (2004) introduce risk in the long-run - a highly persistent variation in expected consumption growth - to explain puzzles in financial markets. The economic channels that drive financial markets are fluctuations in the long-run growth prospects of the economy and economic uncertainty through consumption volatility. These dynamics, together with Epstein and Zin's preferences, successfully explain key asset markets phenomena, namely, equity premium puzzle, asset price volatility puzzle, and return predictability. In the model, financial markets dislike economic uncertainty and better long-run growth prospects raise asset prices. In the foreign exchange market, Colacito and Croce (2011, 2013), and Bansal and Shaliastovich (2013) extend the long-run risk model to a two-country setup. The model is also successful to explain issues in the international financial markets, e.g. the UIP puzzle, and the Backus and Smith anomaly concerning the low correlation between consumption differentials and exchange rates.

Based on the success of the long-run risk model in explaining UIP puzzle, I employ the model as in Bansal and Yaron (2004) and Bansal and Shaliastovich (2013)'s long-run risk model to study the behavior of the exchange rate and the risk premium. The distinction of the model is that the consumption process is specified to contain a small persistence component (long-run risk) with stochastic volatility. Then I study the effect of the stochastic volatilities or risks on the exchange rate and the currency risk premium. Risks in the model are, for example, short- and long-run consumption risk, and monetary policy risk.

Fluctuations in nominal interest rate differentials primarily account for fluctuations in time-varying risk premium. If the nominal interest rate is driven by monetary policy, it calls for the theory of how the policy changes affect the risk premium. Therefore, unlike previous literature on the long-run risk, I address the specification of monetary policy through interest rate in terms of

the Taylor rule. This allows me to investigate how each of the policy specifications could affect the exchange rate and the risk premium. I focus on three rules. The first one is the standard Taylor rule that the nominal interest rate is described by inflation and long-run risk. I called it “Basic Rule”. The second one augments the first one by adding the lag of interest rate so that the interest rate depends also on the past value. This is an “Interest Rate Smoothing Rule”. The last one is the basic rule expanded to add the exchange rate. The “Exchange Rate Rule” is a modified Taylor rule incorporating the exchange rate to account for an open-economy context. The impulse-response analysis reveals that the specification of the interest rate rule matters for a magnitude change of the variables of interest in response to shocks in the model but it does matter for a directional change of these variables.

I follow Backus et al. (2010) procedure to relate the stochastic discount factor to the exchange rate, the interest rate, and the risk premium. I firstly derive an analytic expression for the exchange rate depreciation, interest rate and risk premium; and show that the model can produce a negative UIP coefficient which suggests a deviation from UIP. Theoretically, violation of UIP is explained by the presence of the stochastic volatility of short-run, long-run consumption, and monetary-policy shock volatility.

According to the impulse-response analysis, I find that a short-run shock could deviate a depreciation rate and risk premium for a short period of time, whereas a long-run shock would have more persistent impact on those variables. The responses to changes in level shocks and volatility shocks are different. A decrease in short- and long-run consumption growth level shocks, and an increase in short- and long-run consumption growth volatility shocks induce the exchange rate to appreciate. Regarding the currency risk premium, a change in level shock in short- and long-run consumption growth has no impact on the excess return on foreign deposits. This is possible because the effect of shocks on the exchange rate depreciation and the interest rate differential is cancelling out. A consumption growth level shock alters the depreciation rate, while the interest rate differential automatically adjusts according to the interest rate rule. However, an increase in a volatility shock would raise the excess return on foreign deposits. An agent is risk-averse and dislikes a fall in

economic growth or an increase in uncertainty as they reduce the equilibrium consumption and asset prices. This so contributes to a decrease in price of the foreign currency, inducing the home currency to appreciate. Also, it requires a positive premium to hold a risky currency to compensate high consumption growth risk in the markets. In terms of monetary policy, an increase in policy shock causes an appreciation of the exchange rate and a deviation from UIP in the form of positive currency risk premium.

The organization of this chapter is as follows. In the next section, I document related literature about long-run risk. Then I discuss monetary policy as represented by the Taylor rules, and the relationship between the stochastic discount factor, the exchange rate, and the currency risk premium. In section (3.3), I detail the economic model, then present the solution of the model and discuss its theoretical result in regards to the UIP puzzle. Section (3.4) explains the calibration method and the parameterization of the model. I also show the ability of the model to replicate the population moments of the main variables. The analysis of the impulse-response is performed in this section. I also discuss how the main variables are affected by the shocks. Lastly, conclusions are laid out in section (3.5).

3.2 Related Literature

3.2.1 Long-Run Risk Literature

There are many features in the asset markets that are puzzling from a theoretical perspective. For example, the equity premium puzzle by Mehra and Prescott (1985) finds it is hard to justify a large risk premium with a low risk-free rate. The literature on variance bounds, Shiller (1981), also encounters a problem justifying an excessive market volatility. In the foreign exchange market, Fama (1984) shows evidence of the violation of UIP. Given these difficulties, researchers have attempted to solve the puzzles.

The studies concerned with risk premium can be categorized into two branches. Research in the first branch, e.g. Alvarez and Jermann (2001), and Alvarez et al. (2009), examines a limited-participation model in which the consumption of the marginal investor is variable while the

aggregate consumption is essentially constant. Since the aggregate consumption is constant, it has no role in determining pricing risk. The risk is then priced by the marginal investor. Variation in risk over time is important to investigate movements in asset prices. In their model, the asset market is segmented in the sense that only a fraction of the model's agents choose to participate in the market. The risk premium is varying over time because the degree of market segmentation varies over time and responses endogenously to stochastic shocks. The other branch of research plays with utility functions that produce the marginal utility of consumption to be sensitive to small variations in consumption. Since the actual consumption data contains only small variation, a representative agent in a model with standard utility functions cannot account for large and variable risk premiums. The work of Bansal and Yaron (2004) employs Epstein and Zin (1989) preferences in the long-run risk model to explain key asset market phenomena. Their model can justify the equity premium puzzle, high asset price volatility, and return predictability. Verdelhan (2010) applies the consumption model with external habit formation to explain the counter-cyclical risk premium, and the model simulation results provide successful match with key stylized facts in the asset market. My paper is a part of the second branch, and the model is developed based on the long-run risk model that adopts the Epstein and Zin preferences, which is more like the seminal work of Bansal and Yaron (2004) (hereafter BY).

The idea of the long-run risk model begins from the observation that the drift in log consumption growth is itself a highly persistent stationary process with low conditional volatility but high unconditional volatility. BY call this component long-run risk. In the model, they represent this risk by the conditional mean of consumption growth because it is not constant but volatile over time. BY mimic the specification for consumption to be consistent with observed consumption data. In their model the main economic channels that drive financial markets are fluctuations in the long-run growth prospects of the economy, i.e. consumption growth and dividend growth rates, and the level of economic uncertainty, i.e. consumption volatility.

In the first channel, consumption growth is specified to contain a small persistent expected consumption growth rate component (long-run risk component). This component captures long-run

risk because current shocks to expected growth change expectations about future economic growth not only for short horizons but also for the very long run. The persistence in the expected growth is important as it affects the volatility of asset prices and also determines the risk premium. To account for a large risk premium and volatility of asset prices, BY show that the persistence in expected growth rate has to be large, close to 0.98. The second channel involves time varying economic uncertainty. If consumption growth residuals were *i.i.d.*, the scaled long run variance of consumption or the variance ratio would be flat across different horizon. Instead, BY show that the variance ratio of realized consumption increases with time horizon. That is, agents face a larger aggregate consumption volatility at longer horizons. This increase in aggregate consumption volatility requires a sizeable compensation if the agents prefer early resolution of uncertainty about the consumption path. To allow for time-varying risk premia, the conditional volatility of consumption is modeled to be stochastic, which is distinct from models where growth rates are *i.i.d.* Fluctuations in the conditional volatility of consumption directly affect asset prices. Since a rise in uncertainty leads to a fall in asset prices, shocks to consumption volatility involve a positive risk premium. According to this specification of the economy growth rate, agents dislike a decrease in the long-run growth or an increase in volatility components that will lower consumption, wealth and asset prices. This makes holding equity quite risky. Therefore, it needs to have high risk compensation in equity market. They also show that their specification for consumption growth, which incorporates the high persistent component and whose conditional volatility is time-varying, generates results that are consistent with the real world data and helps justifying many puzzling aspects in the asset market.

Epstein and Zin (1989) preferences play a critical role in the model. While in standard utility functions, there is a one-to-one relationship between the degree of risk aversion and the intertemporal elasticity of substitution (IES), Epstein and Zin preferences disentangle those two parameters. In the BY model, agents prefer an early resolution of uncertainty; that is, the risk aversion is larger than the reciprocal of the IES. Specifically, the degree of risk aversion is around 10 and the IES are larger than 1. For a large degree of risk aversion, agents dislike shocks in the long-run and require a

large premium for bearing those risks. An IES greater than 1 is critical for capturing the observed negative correlation between consumption volatility and price-dividend ratios, and are required for the long run and volatility risks to carry to a positive risk premium.

There is controversy about the existence of the long-run risk component in consumption growth. Observed consumption growth seems to be *i.i.d.*. This implies that any predictable variation component is small and difficult to detect. However, BY believe that only a small component is enough. And it can be shown that the long-run consumption risk can have quantitatively important implication on asset prices if investors have recursive preferences.

Many studies employ different techniques to show for the existence of a long-run component in consumption. BY choose parameters to match the annual moments of consumption and dividend growth. They characterize the equilibrium stochastic discount factor, the short- and long-run market prices, and volatility risks. The model matches the low risk-free rate, and explains the equity premium puzzle and the high asset price volatility. They also show that consumption volatility is an important source of systematic risk that relates to a time-varying risk premium. Bansal et al. (2005) also find that the negative relation between consumption volatility and asset prices (an increase in consumption volatility lowers price-dividend ratios) is robust, which is highlighted by the IES greater than 1 in the model. They also confirm that consumption volatility is an important risk channel. Bansal et al. (2007) test the model using the efficient and generalized method of moments, and find the support of the long-run risk model. Bansal et al. (2012) apply a vector autoregression (VAR) to show that consumption growth is highly predictable at both short- and long-horizons in the long-run risk model. Their results support the view that there is a small long-run predictable component in consumption growth, and consumption volatility is time-varying. Besides, the long-run risk model can account for key properties in the asset market. The work by Hansen et al. (2008) demonstrates how long-run risks are priced in cash flows valuation. A VAR model is used to identify the macroeconomic shocks to be priced. Hansen and Sargent (2007) show that the posterior distribution of a representative consumer behave in a way that supports a long-run risk model, not an *i.i.d.* consumption model.

In a one-country model, consumption growth is not volatile enough to explain the excess returns. In a two-country model, consumption growths do not covary enough to capture returns and exchange rate movements. Colacito and Croce (2011) extend the long-run risks model to a two-country setup to explain international risk sharing and exchange rate volatility. With cross-country-correlated long-run risks and Epstein and Zin preferences, there is a link between long-run growth prospects and exchange rate movements. The way exchange rate enters the model is through a relationship where the growth rate of exchange rate would equal the difference of the log stochastic discount factors for foreign and home currency. Ultimately, the dynamics of the exchange rate is driven by the short- and long-run shocks to international consumption. In the economy, agents fear about uncertainty in the long-run perspective of the economy because shocks to future consumption growth affect their marginal utility of consumption today. They also find that the long-run components of consumption growth are highly persistent and highly correlated across countries. Their correlation increases over time as the volatility of exchange rate growth decreases. The model successfully explains international finance puzzles, for instance, the high correlation across international stock markets despite the lack of correlation of fundamentals.

Colacito and Croce (2013) extend the model into general equilibrium with a two-country, two-good economy with Epstein and Zin preferences and correlated long-run growth prospects. They show that the model can account for the Backus–Smith anomaly, that there is a low correlation between consumption differentials and the exchange rate, and the forward premium anomaly in which the high interest rate currency tends to appreciate. Bansal and Shaliastovich (2013) also develop two-country long-run risks model with time-varying volatilities of expected growth and inflation. The model can capture the violation of the uncovered interest rate parity. In their model, risk premium is driven by the volatilities of expected growth and expected inflation. In high volatility periods, interest rates are low and the risk premium on foreign bonds is high. Due to the volatility channel, exchange rates are predicted by the interest rate differentials, in that high interest rate currency is expected to appreciate.

Colacito et al. (2017) provide empirical evidence that there is strong heterogeneous exposure to global growth news shocks. They developed the model under a framework of multiple countries, and multiple consumption goods whose supplies relate both global and local short- and long-run shocks. They find that Colacito and Croce (2013)'s model fails to replicate the risk premium in the cross section of interest rate–sorted currencies when the long-run shocks are homogeneous exposure to global shock. Thus, they introduce heterogeneous exposure to growth news shocks in the cross section of countries in a way that is consistent with their empirical evidence. These heterogeneous shocks capture a mix of fundamental differences across countries such as size, commodity intensity, monetary policy rules, and financial development. However, none of the long-run risk literature relates monetary policy and the deviation from UIP.

3.2.2 Monetary Policy Represented by Interest Rate Rules

Backus et al. (2010) investigate how monetary policy affects the stochastic discount factor, and how the exchange rate derived from the stochastic discount factor explains the UIP puzzle. The specification of the Taylor rule they considered is

$$\begin{aligned} i_t &= \tau + \tau_\pi \pi_t + \tau_x x_t, \\ i_t^* &= \tau^* + \tau_\pi^* \pi_t^* + \tau_x^* x_t^*, \end{aligned}$$

where $\tau, \tau_\pi, \tau_x, \tau^*, \tau_\pi^*, \tau_x^*$ are policy parameters,¹ i_t is nominal interest rate, π_t is inflation rate, and x_t is consumption growth. Variables with asterisk are for the foreign counterparts. They find that when foreign monetary policy is relatively procyclical, as $\tau_x < \tau_x^*$; and foreign policy being relatively accommodative to inflation, as $\tau_\pi > \tau_\pi^*$, the foreign currency is risky and it needs to compensate the risk by having positive expected excess returns.

The authors show that the UIP violation, indicated by a negative UIP coefficient, relies on the Epstein and Zin preferences parameters in which agents prefer early resolution of uncertainty (the coefficient of risk aversion is greater than 1 and is larger than the reciprocal of the IES). They claim that, however, this conclusion is not a general feature of this Taylor rule. The conclusion about

¹ $\tau_\pi, \tau_x, \tau_\pi^*$, and τ_x^* are positive. τ and τ^* can be either positive or negative.

the UIP coefficient also depends on the rule implied in the model. For example, for a Taylor rule with interest rate smoothing, in which the interest rate depends on both current and past values, a negative UIP coefficient can be achieved if volatility of the policy shock is less autocorrelated than the value of interest rate smoothing policy parameter. They elaborate that some form of the Taylor rule cannot deliver the UIP coefficient to be negative. In their paper, they use a simple setting rule where the nominal interest rate relates to the inflation and consumption growth to articulate their main point about monetary policy and risk premium. Benigno et al. (2012) also point out that the rule depending solely on the inflation gap cannot deliver a negative UIP coefficient; however, the rule with interest rate smoothing can. This is because the negative dependence on the lagged interest rate would reduce the UIP coefficient, and finally turns it negative.

The simple Taylor rule, i.e., that the central bank adjusts interest rates in response to deviations in inflation from its target and to fluctuations in the output gap, is often used to implement monetary policy in closed economies. In an open economies context, the exchange rate is an important part in the monetary policy transmission. For instance, the exchange rate relates to the interest rate by UIP condition. Changes in the exchange rate affect the relative price between domestic and foreign goods and the flow of exports and imports. They also affect the price of foreign goods sold in another country and pass through domestic prices. However, the equilibrium exchange rate is difficult to observe by the central bank. It is challenging for policymakers to implement a policy that is reasonably robust to the specification of the exchange rate model. In this study, I would like to examine the role of the exchange rate in the monetary policy rule. I hence consider the simple Taylor rule that incorporates the exchange rate, and study how the policy accounts for the currency risk premium.

There are several works that study the Taylor rule including the exchange rate. Yet there is controversy about the performance of adding exchange rate to the simple Taylor rule in order to improve economic stability. Ball (1999) studies the Taylor rule that accounts for exchange rate, and finds that an appreciation of the exchange rate would call for monetary policy easing. The rule leads to a better performance measured in terms of the size of real GDP fluctuations around potential

GDP and the size of inflation fluctuations around the target. Considering a similar rule, Svensson (2000) finds that such rule only reduces the inflation variance from 2.1% to 1.8% but increases the output variance from 1.7% to 1.8%. The work of Benigno and Benigno (2001), Batini et al. (2003), and Leitimo and Söderström (2005) also find very small improvements from including exchange rate in the Taylor rule. Taylor (2001) points out that the simple Taylor rule does have an indirect effect of interest rate to the exchange rate since inflation and output are strongly affected by the exchange rate. A separate response of the exchange rate will cause only marginal improvements. Nevertheless, Froyen and Guender (2016) argue that the view that including the exchange rate has either small or negative performance should be re-assessed since an increase in the instability in world financial market would induce the central bank to add exchange rate stability to the list of policy goals. They find that a small weight on real exchange rate stability in the loss function is sufficient to improve the Taylor rule performance relative to the optimal policy. A central bank that values real exchange rate stability and follows the Taylor rule should response to the real exchange rate because it helps reducing relative losses.

3.2.3 Stochastic Discount Factor, Exchange Rate and Uncovered Interest Rate Parity

One of the most distinct theorems in asset pricing states that in the absence of arbitrage opportunities, there exists a stochastic discount factor, N_{t+1} such that the rate of return on any asset j denominated in units of home currency, denoted by $R_{j,t}$, satisfies $E_t(N_{t+1}R_{j,t+1}) = 1$. Similarly, $E_t(N_{t+1}^*R_{j,t+1}^*) = 1$ for all foreign-currency denominated returns. The returns of foreign asset expressed in units of home currency satisfy $E_t(N_{t+1}\frac{S_{t+1}}{S_t}R_{j,t+1}^*) = 1$, where S_t is the level of (nominal) spot exchange rate expressed in units of home currency per unit of foreign currency.

Assuming that asset markets are complete, the log growth rate of exchange rate can be expressed in terms of the difference between log of foreign and log of home stochastic discount factors

$$\Delta s_{t+1} = n_{t+1}^* - n_{t+1}, \quad (3.1)$$

where the small letters s_t , n_{t+1}^* , and n_{t+1} is the logarithm of S_t , N_{t+1}^* , and N_{t+1} , respectively.

Define N_{t+1} as a nominal stochastic discount factor and M_{t+1} as a real stochastic discount factor. The nominal stochastic discount factor is defined as $N_{t+1} = M_{t+1} \exp(-\pi_{t+1})$. The relationship between nominal and real variables can be expressed in terms of exchange rate depreciation as follows:

$$\Delta s_{t+1} = n_{t+1}^* - n_{t+1} = (m_{t+1}^* - m_{t+1}) - (\pi_{t+1}^* - \pi_{t+1}), \quad (3.2)$$

where $m_{t+1} = \log(M_{t+1})$, and π is the home inflation. The nominal risk-free interest rate relates the stochastic discount factor as follows:

$$i_t = -\log E_t N_{t+1}. \quad (3.3)$$

The UIP states that the return on a home deposit will equal to the return on a foreign deposit adjusted for the exchange rate. This implies that $s_{t+1} - s_t = i_t - i_t^*$. Fama's (1984) regression represents the UIP by

$$s_{t+1} - s_t = a + b(i_t - i_t^*) + \epsilon_{t+1},$$

where ϵ_t is the regression residual. The UIP coefficient, denoted by b , can be written in terms of

$$b = \frac{\text{cov}(\Delta s_{t+1}, i_t - i_t^*)}{\text{var}(i_t - i_t^*)}. \quad (3.4)$$

If UIP holds, the estimated slope coefficient, b , should equal to 1. Nevertheless, the estimated slope coefficient is found to be negative which implies the violation of UIP. This suggests that high interest rate currency tends to appreciate.

The expected excess return on the foreign deposits or the risk premium, denoted by $\rho_t = E_t \Delta s_{t+1} - i_t + i_t^*$, can be written as follows

$$\rho_t = \frac{1}{2} [\text{var}_t(\log N_{t+1}) - \text{var}_t(\log N_{t+1}^*)]. \quad (3.5)$$

ρ_t captures the deviation from UIP, which would happen if an investor is risk averse and demands a premium to hold foreign currency. With a positive risk premium, high interest rate currencies tend to appreciate over time. Equation (3.5) says that the country with low variability of the stochastic discount factor or the pricing kernel has a relatively risky currency.

3.3 Model Specification

In this section, I present a model in which the risk in the long-run and monetary policy volatility affect the deviation from UIP. The long-run risk model is based on Bansal and Yaron (2004). The distinct feature of the model is that the consumption growth path is specified exogenously to contain (1) a small long-run component, and (2) fluctuating economic uncertainty (consumption volatility). In this way, the consumption prospect is defined to fit the observed consumption facts. As common in the long-run risk literature, I employ Epstein and Zin preferences with the specification of an early resolution of uncertainty. The deviation from the UIP can be generated by the presence of both consumption stochastic volatility and the Epstein and Zin preferences. I follow Backus et al. (2010) to account for the monetary policy in the model. Inflation is determined by a central bank that uses the nominal interest rate as its instrument. I work with an endowment economy in which each country receives a stochastic endowment at each point in time. In equilibrium, the consumption demand equals the given supply. Since the consumption path is specified exogenously, the model is silent about international trade that gives rise to such consumption allocations. However, the advantage of working with an the endowment economy is that I can derive an analytical formula for the risk premium, the UIP coefficient, and the exchange rate depreciation.

3.3.1 The Model

3.3.1.1 Preferences and Stochastic Discount Factor

There are two countries: home and foreign. A home country representative consumer maximizes a utility function with the Epstein and Zin (1989) preferences given by

$$U_t = [(1 - \beta)c_t^\rho + \beta\mu_t(U_{t+1})^\rho]^{1/\rho}, \quad (3.6)$$

where β is time preference, and $1/(1 - \rho)$ is the intertemporal elasticity of substitution (IES). The certainty equivalence of random future utility is

$$\mu_t(U_{t+1}) = E_t[U_{t+1}^\alpha]^{1/\alpha}, \quad (3.7)$$

where $1 - \alpha$ is the coefficient of relative risk aversion. The magnitudes of α and ρ determine whether agent prefers early ($\alpha < \rho$) or late resolution of uncertainty ($\alpha > \rho$). Standard CRRA preference characterizes $\alpha = \rho$. At each date t , the agent trades a complete set of N assets in the market. The budget constraint is

$$c_t + \sum_{i=1}^N P_{it}A_{i,t+1} = \sum_{i=1}^N d_{it}A_{it}, \quad (3.8)$$

where A_{it} is the quantity of asset i that the representative agent holds at time t , P_{it} is the price of 1 unit of asset i at time t , and d_{it} is the payoff per 1 unit of asset i at time t .

The marginal rate of intertemporal substitution, denoted by M_{t+1} , is defined as

$$M_{t+1} = \beta \left(\frac{c_{t+1}}{c_t} \right)^{\rho-1} \left(\frac{U_{t+1}}{\mu_t(U_{t+1})} \right)^{\alpha-\rho}. \quad (3.9)$$

M_{t+1} is referred as a real stochastic discount factor or a real pricing kernel. If the utility maximization problem is formulated in nominal term, the nominal stochastic discount factor is N_{t+1} . And, it can be expressed as follows

$$N_{t+1} = M_{t+1} \exp(-\pi_{t+1}), \quad (3.10)$$

where π_{t+1} is the rate of inflation. The foreign country representative agent has preferences and budget constraint defined analogously with the asterisks sign denoting foreign variables.

International financial markets are assumed to be complete. Agents have access to a complete set of contingent securities that are traded internationally. With complete markets, the stochastic discount factor is unique and satisfies the following expression:

$$\frac{S_{t+1}}{S_t} = \frac{N_{t+1}^*}{N_{t+1}} = \frac{M_{t+1}^* \exp(-\pi_{t+1}^*)}{M_{t+1} \exp(-\pi_{t+1})}. \quad (3.11)$$

3.3.1.2 Consumption Prospects

Define domestic consumption growth as $x_{t+1} \equiv \log(c_{t+1}/c_t)$. The consumption growth process is exogenously modeled to contain a small and persistent long-run component with stochastic volatility. The process is defined as follows:

$$x_{t+1} = \mu + z_t + \sqrt{u_t} \varepsilon_{t+1}^x, \quad (3.12)$$

where μ is the mean of consumption growth, u_t is the conditional volatility of consumption growth, and ε_{t+1}^x is a standard normal shock. The term $\sqrt{u_t} \varepsilon_{t+1}^x$ is called the short-run shock, so that u_t can be called the variance of the short-run shock.

z_t is the small long-run component in the expected of consumption growth. The dynamics of z_t is governed by

$$z_{t+1} = \varphi_z z_t + \sqrt{w_t} \varepsilon_{t+1}^z, \quad (3.13)$$

where φ_z is a parameter representing the persistence of expected consumption growth, w_t is the conditional volatility of the expected consumption growth, and ε_{t+1}^z is a standard normal shock. The term $\sqrt{w_t} \varepsilon_{t+1}^z$ is called the long-run shock. z_t captures small, persistent time variation in the conditional expectation of consumption growth as $E_t(x_{t+1}) = \mu + z_t$. It is called long-run risk because it captures the long-run variation in expected consumption growth. Since z_t is AR(1), I can write z_t in terms of the sum of all long-run shocks ($z_t = (1 + \varphi_z L + \varphi_z^2 L^2 + \dots) \sqrt{w_{t-1}} \varepsilon_t^z$), where L is a lag operator.

The volatility of consumption growth, and the volatility of the expected consumption growth are modeled to be stochastic. The stochastic volatilities of u_t and w_t are defined in the following way:²

$$u_{t+1} = (1 - \varphi_u)\theta_u + \varphi_u u_t + \sigma_u \varepsilon_{t+1}^u, \quad (3.14)$$

$$w_{t+1} = (1 - \varphi_w)\theta_w + \varphi_w w_t + \sigma_w \varepsilon_{t+1}^w, \quad (3.15)$$

where φ_u is the autocorrelation of short-run volatility, φ_w is the autocorrelation of long-run volatility, θ_u is the mean of short-run volatility, θ_w is the mean of long-run volatility, σ_u is the volatility of short-run volatility, and σ_w is the volatility of long-run volatility. The volatility shocks are assumed to be multivariate normal and independent within-country ($\varepsilon^x, \varepsilon^z, \varepsilon^u, \varepsilon^w$) $\sim NID(0, I)$, but allowed for correlation across countries: $\eta_j \equiv \text{corr}(\varepsilon^j, \varepsilon^{j*})$, for $j = (x, z, u, w)$.

The main idea of the long-run risk model is that current shocks to expected consumption growth alter expectations about future economic growth not only for short horizons but also for the very long run. Moreover, the conditional volatility of consumption is time-varying. Fluctuations in consumption volatility contribute to time variation in the risk premium. The model accounts for three sources of risks that determine the risk premium which are short-run risk, long-run risk, and consumption volatility risk. Agents fear movements in the long-run growth and volatility components because they affect the equilibrium consumption and asset prices. Thus, agents demand compensation for holding risky assets.

On the supply side, each country receives a stochastic endowment y_t at each point in time. The feasibility constraint of each country satisfies $c_t = y_t$. In this endowment economy, consumption prospects are defined exogenously but in equilibrium consumption demand equates to the given supply.

²Volatilities u_t and w_t must be positive but stochastic volatilities defined in this way can give negative values. When I solve the model numerically, I ensure that these volatilities are positive by replacing negative realizations with zeros. This happens for about 5% of the realizations. For example, this is done by choosing σ_u^2 , given values of θ_u and φ_u , to be as large as possible subject to the constraint that the probability of observing a negative realization of u_t does not exceed 5%. This specification of volatilities are standard in the long-run risk literature. Moreover, assuming this form allows to solve for the closed-form solutions of the interest rate differential, the exchange rate depreciation, and the currency risk premium.

3.3.1.3 Monetary Policy

In this section, I will focus on the interest rate rule that depends on inflation, expected consumption growth, and exchange rate. The nominal interest rates in the home and foreign countries follow the Taylor rule of the form:³

$$i_t = \tau + \tau_\pi \pi_t + \tau_z z_t + \tau_s \Delta s_t + v_t, \quad (3.16)$$

$$i_t^* = \tau^* + \tau_\pi^* \pi_t^* + \tau_z^* z_t^* + \tau_s^* \Delta s_t^* + v_t^*, \quad (3.17)$$

where v_t and v_t^* are policy shocks, and $\tau, \tau_\pi, \tau_z, \tau_s, \tau^*, \tau_\pi^*, \tau_z^*, \tau_s^*$ are policy parameters. The policy shocks are assumed to be stochastic in the following way:

$$v_{t+1} = \varphi_v v_t + \sqrt{e_t} \varepsilon_{t+1}^v, \quad (3.18)$$

$$e_{t+1} = (1 - \varphi_e) \theta_e + \varphi_e e_t + \sigma_e \varepsilon_{t+1}^e, \quad (3.19)$$

$$v_{t+1}^* = \varphi_v^* v_t^* + \sqrt{e_t^*} \varepsilon_{t+1}^{v^*}, \quad (3.20)$$

$$e_{t+1}^* = (1 - \varphi_e^*) \theta_e^* + \varphi_e^* e_t^* + \sigma_e^* \varepsilon_{t+1}^{e^*}, \quad (3.21)$$

where φ_v and φ_v^* are the autocorrelation of policy shocks. e_t and e_t^* are stochastic volatilities of policy shocks, with autocorrelation φ_e and φ_e^* , and mean θ_e and θ_e^* .

The Taylor rules in equations (3.16) and (3.17) are extended from the basic specification. First, the nominal interest rates are determined by the expected consumption growth, instead of the output gap as in typical rules. The distinction is insignificant because the model has no friction that can give rise to a gap. Second, the rules have included exchange rates and policy shocks so that I could study how policy shocks and exchange rates affect the risk premium.

3.3.2 Solving the Model

To derive an analytical solution for the risk premium, I follow the Backus et al. (2010) procedure. I firstly find the linear approximation of the real stochastic discount factor (sdf) or the pricing kernel.

³For parsimony, I use the expected consumption growth z_t instead of its current level x_t as in standard literature. Doing so reduces the number of state variable by one variable.

Then I find the approximation for the nominal parts. The approximation of the log of real sdf in equation (3.9) is⁴

$$-m_{t+1} = \delta^r + \gamma_z^r z_t + \gamma_u^r u_t + \gamma_w^r w_t + \lambda_x^r \sqrt{u_t} \varepsilon_{t+1}^x + \lambda_z^r \sqrt{w_t} \varepsilon_{t+1}^z + \lambda_u^r \sigma_u \varepsilon_{t+1}^u + \lambda_w^r \sigma_w \varepsilon_{t+1}^w, \quad (3.22)$$

where $\delta^r = [-\log(\beta) + (1-\rho)\mu + \frac{\alpha}{2}(\alpha-\rho)]$, $\gamma_z^r = 1-\rho$, $\gamma_u^r = \frac{\alpha}{2}(\alpha-\rho)$, $\gamma_w^r = \frac{\alpha}{2}(\alpha-\rho)\omega_z^2$, $\lambda_x^r = 1-\alpha$, $\lambda_z^r = -(\alpha-\rho)\omega_z$, $\lambda_u^r = -(\alpha-\rho)\omega_u$, $\lambda_w^r = -(\alpha-\rho)\omega_w$. The real sdf depends on the expected consumption growth, the stochastic volatilities, the short- and long-run shocks, and the stochastic volatility shocks.

Consider the Taylor rules in equations (3.16) and (3.17), the implied inflation equations are

$$\pi_t = \frac{1}{\tau_\pi} [i_t - \tau - \tau_z z_t - \tau_s \Delta s_t - v_t], \quad (3.23)$$

$$\pi_t^* = \frac{1}{\tau_\pi^*} [i_t^* - \tau^* - \tau_z^* z_t^* - \tau_s^* \Delta s_t^* - v_t^*]. \quad (3.24)$$

I conjecture the inflation solution in these forms:

$$\pi_t = a + a_1 z_t + a_2 z_t^* + a_3 u_t + a_4 u_t^* + a_5 w_t + a_6 w_t^* + a_7 v_t + a_8 v_t^* + a_9 e_t + a_{10} e_t^*, \quad (3.25)$$

$$\pi_t^* = a^* + a_1^* z_t + a_2^* z_t^* + a_3^* u_t + a_4^* u_t^* + a_5^* w_t + a_6^* w_t^* + a_7^* v_t + a_8^* v_t^* + a_9^* e_t + a_{10}^* e_t^*. \quad (3.26)$$

Using the fact that the nominal interest rate relates to the nominal sdf in the following way:

$$\begin{aligned} i_t &= -\log E_t(N_{t+1}) \\ &= -\log E_t(\exp(\log M_{t+1} - \pi_{t+1})). \end{aligned} \quad (3.27)$$

Collecting terms for $\log M_{t+1}$ and π_{t+1} , I can derive for the nominal interest rate. Then I substitute the nominal interest rate into equation (3.23), and solve for the a_j coefficients by matching up the results with the conjecture solution for inflation. The process for foreign country is done in the same way. The expressions for the a_j and a_j^* coefficients are shown in Appendix C.2.

Next, I derive for the expression of the nominal sdf. The linearized nominal sdf is

$$\begin{aligned} -n_{t+1} &= \delta + \gamma_z z_t + \gamma_z^* z_t^* + \gamma_u u_t + \gamma_u^* u_t^* + \gamma_w w_t + \gamma_w^* w_t^* + \gamma_v v_t + \gamma_v^* v_t^* + \gamma_e e_t + \gamma_e^* e_t^* + \\ &\quad \lambda_x \sqrt{u_t} \varepsilon_{t+1}^x + \lambda_z \sqrt{w_t} \varepsilon_{t+1}^z + \lambda_z^* \sqrt{w_t^*} \varepsilon_{t+1}^{z^*} + \lambda_u \sigma_u \varepsilon_{t+1}^u + \lambda_u^* \sigma_u^* \varepsilon_{t+1}^{u^*} + \lambda_w \sigma_w \varepsilon_{t+1}^w + \\ &\quad \lambda_w^* \sigma_w^* \varepsilon_{t+1}^{w^*} + \lambda_v \sqrt{e_t} \varepsilon_{t+1}^v + \lambda_v^* \sqrt{e_t^*} \varepsilon_{t+1}^{v^*} + \lambda_e \sigma_e \varepsilon_{t+1}^e + \lambda_e^* \sigma_e^* \varepsilon_{t+1}^{e^*}, \end{aligned} \quad (3.28)$$

⁴The calculation is shown in Appendix C.1

where $\delta = \delta^r + a + a_3(1 - \varphi_u)\theta_u + a_4(1 - \varphi_u^*)\theta_u^* + a_5(1 - \varphi_w)\theta_w + a_6(1 - \varphi_w^*)\theta_w^* + a_7(1 - \varphi_v)\theta_v + a_8(1 - \varphi_v^*)\theta_v^* + a_9(1 - \varphi_e)\theta_e + a_{10}(1 - \varphi_e^*)\theta_e^*$, $\gamma_z = \gamma_z^r + a_1\varphi_z$, $\gamma_z^* = a_2\varphi_z^*$, $\gamma_u = \gamma_u^r + a_3\varphi_u$, $\gamma_u^* = a_4\varphi_u^*$, $\gamma_w = \gamma_w^r + a_5\varphi_w$, $\gamma_w^* = a_6\varphi_w^*$, $\gamma_v = a_7\varphi_v$, $\gamma_v^* = a_8\varphi_v^*$, $\gamma_e = a_9\varphi_e$, $\gamma_e^* = a_{10}\varphi_e^*$, $\lambda_x = \lambda_x^r$, $\lambda_z = \lambda_z^r + a_1$, $\lambda_z^* = a_2$, $\lambda_u = \lambda_u^r + a_3$, $\lambda_u^* = a_4$, $\lambda_w = \lambda_w^r + a_5$, $\lambda_w^* = a_6$, $\lambda_v = a_7$, $\lambda_v^* = a_8$, $\lambda_e = a_9$, $\lambda_e^* = a_{10}$.

The nominal sdf for the foreign country can be written as:

$$\begin{aligned} -n_{t+1}^* &= \delta^* + \gamma_z^* z_t + \gamma_z^{*z} z_t^* + \gamma_u^* u_t + \gamma_u^{*u} u_t^* + \gamma_w^* w_t + \gamma_w^{*w} w_t^* + \gamma_v^* v_t + \gamma_v^{*v} v_t^* + \gamma_e^* e_t + \gamma_e^{*e} e_t^* + \\ &\lambda_x^* \sqrt{u_t^*} \varepsilon_{t+1}^{*x} + \lambda_z^* \sqrt{w_t} \varepsilon_{t+1}^{*z} + \lambda_z^{*z} \sqrt{w_t^*} \varepsilon_{t+1}^{*z} + \lambda_u^* \sigma_u \varepsilon_{t+1}^u + \lambda_u^{*u} \sigma_u^* \varepsilon_{t+1}^{*u} + \lambda_w^* \sigma_w \varepsilon_{t+1}^w + \\ &\lambda_w^{*w} \sigma_w^* \varepsilon_{t+1}^{*w} + \lambda_v^* \sqrt{e_t} \varepsilon_{t+1}^{*v} + \lambda_v^{*v} \sqrt{e_t^*} \varepsilon_{t+1}^{*v} + \lambda_e^* \sigma_e \varepsilon_{t+1}^e + \lambda_e^{*e} \sigma_e^* \varepsilon_{t+1}^{*e}, \end{aligned} \quad (3.29)$$

where $\delta = \delta^{*r} + a^* + a_3^*(1 - \varphi_u)\theta_u + a_4^*(1 - \varphi_u^*)\theta_u^* + a_5^*(1 - \varphi_w)\theta_w + a_6^*(1 - \varphi_w^*)\theta_w^* + a_7^*(1 - \varphi_v)\theta_v + a_8^*(1 - \varphi_v^*)\theta_v^* + a_9^*(1 - \varphi_e)\theta_e + a_{10}^*(1 - \varphi_e^*)\theta_e^*$, $\gamma_z^* = a_1^*\varphi_z$, $\gamma_z^{*z} = \gamma_z^{*r} + a_2^*\varphi_z^*$, $\gamma_u^* = a_3^*\varphi_u$, $\gamma_u^{*u} = \gamma_u^{*r} + a_4^*\varphi_u^*$, $\gamma_w^* = a_5^*\varphi_w$, $\gamma_w^{*w} = \gamma_w^{*r} + a_6^*\varphi_w^*$, $\gamma_v^* = a_7^*\varphi_v$, $\gamma_v^{*v} = a_8^*\varphi_v^*$, $\gamma_e^* = a_9^*\varphi_e$, $\gamma_e^{*e} = a_{10}^*\varphi_e^*$, $\lambda_x^* = \lambda_x^{*r}$, $\lambda_z^* = a_1^*$, $\lambda_z^{*z} = \lambda_z^{*r} + a_2^*$, $\lambda_u^* = a_3^*$, $\lambda_u^{*u} = \lambda_u^{*r} + a_4^*$, $\lambda_w^* = a_5^*$, $\lambda_w^{*w} = \lambda_w^{*r} + a_6^*$, $\lambda_v^* = a_7^*$, $\lambda_v^{*v} = a_8^*$, $\lambda_e^* = a_9^*$, $\lambda_e^{*e} = a_{10}^*$.

Therefore, the home and foreign nominal interest rates can be written as:

$$\begin{aligned} i_t &= \iota + \gamma_z z_t + \gamma_z^* z_t^* + (\gamma_u - (\lambda_x^r)^2/2)u_t + \gamma_u^* u_t^* + (\gamma_w - (\lambda_z^r + a_1)^2/2)w_t + \\ &(\gamma_w^* - (a_2)^2/2)w_t^* + \gamma_v v_t + \gamma_v^* v_t^* + (\gamma_e - (a_7)^2/2)e_t + (\gamma_e^* - (a_8)^2/2)e_t^*, \end{aligned} \quad (3.30)$$

$$\begin{aligned} i_t^* &= \iota^* + \gamma_z^* z_t + \gamma_z^{*z} z_t^* + \gamma_u^* u_t + (\gamma_u^{*u} - (\lambda_x^{*r})^2/2)u_t^* + (\gamma_w^* - (a_1^*)^2/2)w_t + \\ &(\gamma_w^{*w} - (\lambda_z^{*r} + a_2^*)^2/2)w_t^* + \gamma_v^* v_t + \gamma_v^{*v} v_t^* + (\gamma_e^* - (a_7^*)^2/2)e_t + (\gamma_e^{*e} - (a_8^*)^2/2)e_t^* \end{aligned} \quad (3.31)$$

where $\iota = \delta - [(\lambda_x^r + a_3)^2\sigma_u^2 + (\lambda_w^r + a_5)^2\sigma_w^2 + (a_4)^2(\sigma_u^*)^2 + (a_6)^2(\sigma_w^*)^2 + (a_9)^2\sigma_e^2 + (a_{10})^2(\sigma_e^*)^2]/2$, and $\iota^* = \delta^* - [(\lambda_x^{*r} + a_4^*)^2(\sigma_u^*)^2 + (\lambda_w^{*r} + a_6^*)^2(\sigma_w^*)^2 + (a_3^*)^2(\sigma_u)^2 + (a_5^*)^2(\sigma_w)^2 + (a_9^*)^2\sigma_e^2 + (a_{10}^*)^2(\sigma_e^*)^2]/2$.

In the next section, I will make use of these equations to derive for the relationship of the exchange rate and the interest rate.

3.3.3 Model Solutions of Exchange Rate and Currency Risk Premium

In this section, I derive the expression for the expected depreciation, the nominal interest rate differential, and the risk premium. Then, I find the UIP coefficient. These expressions can be written as functions of the state variables z_t, u_t, w_t, v_t, e_t , and these variables with asterisk for the foreign counterparts.

The exchange rate depreciation is the difference between the nominal sdf of the foreign and home countries. The expected depreciation can be derived as follows

$$E_t \Delta s_{t+1} = (\delta - \delta^*) + (\gamma_z - \gamma_z^*)z_t + (\gamma_{z^*} - \gamma_{z^*}^*)z_t^* + (\gamma_u - \gamma_u^*)u_t + (\gamma_{u^*} - \gamma_{u^*}^*)u_t^* + (\gamma_w - \gamma_w^*)w_t + (\gamma_{w^*} - \gamma_{w^*}^*)w_t^* + (\gamma_v - \gamma_v^*)v_t + (\gamma_{v^*} - \gamma_{v^*}^*)v_t^* + (\gamma_e - \gamma_e^*)e_t + (\gamma_{e^*} - \gamma_{e^*}^*)e_t^*. \quad (3.32)$$

The nominal interest rate differential is:

$$\begin{aligned} i_t - i_t^* &= (\iota - \iota^*) + (\gamma_z - \gamma_z^*)z_t + (\gamma_{z^*} - \gamma_{z^*}^*)z_t^* + (\gamma_u - (\lambda_x^r)^2/2 - \gamma_u^*)u_t + \\ &(\gamma_{u^*} - \gamma_{u^*}^* + (\lambda_x^{*r})^2/2)u_t^* + (\gamma_w - (\lambda_z^r + a_1)^2/2 - \gamma_w^* + (a_1^*)^2/2)w_t + \\ &(\gamma_{w^*} - (a_2)^2/2 - \gamma_{w^*}^* + (\lambda_z^{*r} + a_2^*)^2/2)w_t^* + \\ &(\gamma_v - \gamma_v^*)v_t + (\gamma_{v^*} - \gamma_{v^*}^*)v_t^* + (\gamma_e - (a_7)^2/2 - \gamma_e^* + (a_7^*)^2/2)e_t + \\ &(\gamma_{e^*} - (a_8)^2/2 - \gamma_{e^*}^* + (a_8^*)^2/2)e_t^*. \end{aligned} \quad (3.33)$$

Therefore, the risk premium on the foreign currency is:

$$\begin{aligned} \rho_t &= [\lambda_x^2 u_t - (\lambda_x^*)^2 u_t^* + (\lambda_z^2 - (\lambda_z^*)^2)w_t + (\lambda_{z^*}^2 - (\lambda_{z^*}^*)^2)w_t^* + (\lambda_v^2 - (\lambda_v^*)^2)e_t + (\lambda_{v^*}^2 - (\lambda_{v^*}^*)^2)e_t^* + \\ &(\lambda_u^2 - (\lambda_u^*)^2)\sigma_u^2 + (\lambda_{u^*}^2 - (\lambda_{u^*}^*)^2)(\sigma_{u^*}^*)^2 + (\lambda_w^2 - (\lambda_w^*)^2)\sigma_w^2 + (\lambda_{w^*}^2 - (\lambda_{w^*}^*)^2)(\sigma_{w^*}^*)^2 + \\ &(\lambda_e^2 - (\lambda_e^*)^2)\sigma_e^2 + (\lambda_{e^*}^2 - (\lambda_{e^*}^*)^2)(\sigma_{e^*}^*)^2]/2. \end{aligned} \quad (3.34)$$

Given the stochastic volatilities, denoted by u_t, w_t, e_t and its foreign counterpart with asterisk terms. The time variation in currency risk premium, reflects the time-varying compensations for the expected consumption growth risk, is driven by the stochastic volatility of consumption growth (u_t), the stochastic volatility of expected consumption growth (w_t), and the stochastic volatility of

monetary policy shock (e_t). If these volatilities are constant, the excess return on foreign deposit are constant too.

The nominal UIP coefficient can be formulated as:⁵

$$b = \frac{\text{cov}(\Delta s_{t+1}, i_t - i_t^*)}{\text{var}(i_t - i_t^*)} = \frac{\begin{pmatrix} (\gamma_z - \gamma_z^*)^2 \text{var}(z_t - z_t^*) + (\gamma_v - \gamma_v^*)^2 \text{var}(v_t - v_t^*) + \\ (\gamma_u - \gamma_u^*)(\gamma_u - \gamma_u^* - (\lambda_x^r)^2/2) \text{var}(u_t - u_t^*) + \\ (\gamma_w - \gamma_w^*)(\gamma_w - \gamma_w^* - [(\lambda_z^r + a_1)^2 - (a_1^*)^2]/2) \text{var}(w_t - w_t^*) + \\ (\gamma_e - \gamma_e^*)(\gamma_e - \gamma_e^* - [(a_7)^2 - (a_7^*)^2]/2) \text{var}(e_t - e_t^*) \end{pmatrix}}{\begin{pmatrix} (\gamma_z - \gamma_z^*)^2 \text{var}(z_t - z_t^*) + (\gamma_v - \gamma_v^*)^2 \text{var}(v_t - v_t^*) + \\ (\gamma_u - \gamma_u^* - (\lambda_x^r)^2/2) \text{var}(u_t - u_t^*) + (\gamma_w - \gamma_w^* - (\lambda_z^r + a_1)^2/2 + (a_1^*)^2/2) \text{var}(w_t - w_t^*) + \\ (\gamma_e - \gamma_e^* - [(a_7)^2 - (a_7^*)^2]/2) \text{var}(e_t - e_t^*) \end{pmatrix}} \quad (3.35)$$

Without the stochastic volatility, denoted by u_t , w_t , e_t and its foreign counterpart with asterisk terms, the UIP coefficient will equal to one, which implies the UIP holds. With the presence of these stochastic volatilities, the model can produce negative UIP coefficient. Examining the above expression, the UIP coefficient is to be negative if the numerator is negative. The necessary condition is that there is at least one term of coefficients of the variance of stochastic volatility $\text{var}(u_t - u_t^*)$, $\text{var}(w_t - w_t^*)$, and $\text{var}(e_t - e_t^*)$ is negative. That is, in each coefficient of the variance of stochastic volatility, the two terms in parenthesis have opposite signs. For example, the sufficient condition that the coefficient of $\text{var}(u_t - u_t^*)$ is negative requires the early preference of resolution ($\alpha < \rho$) and the degree of risk aversion is greater than 1. Lastly, the sum of the negative term(s) should be greater than the sum of the positive terms.

3.4 Quantitative Results

In this section, I firstly discuss values of calibrated model parameters, then I focus on the ability of the model accounting for a set of sample moments of consumption growth, inflation,

⁵The calculation is based on the assumption that all parameters are symmetric across countries.

exchange rate and interest rate to replicate the actual data. I employ the financial data between two countries: the United Kingdom, and the United States. Data used in the study covers the series of consumption, inflation, short-term interest rate, and exchange rate. The data are quarterly and span from 1970 through 2017. All data series are from the Organization for Economic Cooperation and Development (OEACD). Last but not least, I demonstrate how the risks affect these important variables by means of impulse response analysis.

3.4.1 Calibration of Parameter Values

To begin with, I categorize the parameters into 3 groups: preference parameters, consumption prospect parameters, and interest rate rule parameters. Preference and consumption parameters are for calculated values and moments for the real side of the model, e.g. consumption, real interest rate and real exchange rate. I impose symmetry in preference and consumption process parameters across the two countries, so that the parameter values and real moments are the same across countries even though the shocks are not. These real parameters are to match U.S. data on consumption, U.K. real interest rate, and the real exchange rate between U.K. and U.S. Taylor rule parameters are to match nominal data from the U.K. and U.S.

For preference parameters, I firstly set the intertemporal elasticity of substitution (IES, $(1 - \rho)^{-1}$) to 1.5 as suggested in Bansal and Yaron (2004). The relative risk aversion $(1 - \alpha)$ is chosen to match the variance of real depreciation rate while the discount factor (β) matches with the mean of the U.K. quarterly real interest rates. The discount factor is around 0.997, and the relative risk aversion is 5.8175.

Given the series of the Unites States' consumption data, I calculate the mean of consumption growth (μ) . According to equation (3.12), I apply Hodrick-Prescott (HP) filter to term $(x_{t+1} - \mu)$ to separate trend and cycle components. From the trend component, I take its series as z_t . The cycle component would approximate to $\sqrt{u_t} \varepsilon_{t+1}^x$. I then apply HP filter again to the term $\log(\sqrt{u_t} \varepsilon_{t+1}^x)^2$ to get trend component as $\log(u_t)$, and cycle component. From this point, I could calculate the mean of short-run volatility (θ_u) and autocorrelation of short-run volatility (φ_u) . Since I have the

z_t series estimated from the first filter, I can estimate the autocorrelation of long-run risk (φ_z). Then from equation (3.13), I can figure for $\sqrt{w_{t-1}}\varepsilon_t^z$. I use HP filter on $\log(\sqrt{w_{t-1}}\varepsilon_t^z)^2$ to compute w_t series. Then I compute for the mean and autocorrelation of long-run volatility (θ_w and φ_w , respectively). Applying the same approach to the policy shock (v_t) and policy shock volatility (e_t), I obtain value of parameters relevant to policy shock.

Given values of θ_u and φ_u , the conditional variance of short-run volatility, σ_u is chosen to be as large as possible subject to the constraint the probability of observing a negative realization of u_t at 5%. The same logic applies to the values of σ_w and σ_e in that these conditional variance of volatilities are chosen to observe a negative realization of w_t and e_t at 5%.

When I do a simulation, I generate data for 292 observations including 100 burn-in periods. I will have 192 observations of each time series as the actual data. The 100-period burn-in will ensure that the simulated data is approaching the steady state. Moreover, each time series has been generated 10,000 rounds. I calculate the sample moments of each round, then average them over 10,000 replications. This will ensure the accuracy of sample moments. Table (3.1) reports preferences and consumption parameters.

For Taylor rule parameters, I consider three different rules. The first one is to examine how interest rate responds to inflation and mean of consumption growth. The second rule is the Taylor rule with interest rate smoothing. The interest rate gradually moves so that it depends on its past value. The third rule consists of augmenting the first one with the exchange rate. In each rule, there are two alternatives of the calibration procedure. The first calibration is targeting to match the model with domestic and foreign inflation - interest rate. The second one is targeting the variance of nominal exchange rate depreciation and the UIP coefficient.

Table (3.2) summarizes how nominal parameters are calibrated, and Table (3.3) displays the values of calibrated nominal parameters. The intercept parameters are negative while the coefficient of Taylor rule parameters are all positive as they should be. It is interesting that the inflation coefficients, τ_π , of any rule are very high when matching with exchange rate volatility and UIP slope. Especially for the rule that depends on exchange rate, τ_s^* is tiny and almost zero. This may

Table 3.1: Calibrated Real Parameter Values

Description	Parameter	Value
Preferences:		
IES	$(1 - \rho)^{-1}$	1.5
Relative risk aversion	$1 - \alpha$	5.8175
Discount factor	β	0.9970
Consumption prospect:		
Mean of consumption growth	μ	0.0074
Autocorrelation of long-run risk	φ_z	0.9978
Volatility of consumption growth:		
Mean of short-run volatility	θ_u	3.6003×10^{-5}
Autocorrelation of short-run volatility	φ_u	0.9939
Volatility of short-run volatility	σ_u	9.6821×10^{-7}
Mean of long-run volatility	θ_w	1.3831×10^{-8}
Autocorrelation of long-run volatility	φ_w	0.9995
Volatility of long-run volatility	σ_w	2.3354×10^{-10}
Monetary policy shock:		
Autocorrelation of policy shock	φ_v	0.8053
Mean of policy shock volatility	θ_e	7.8646×10^{-5}
Autocorrelation of policy shock volatility	φ_e	0.9856
Volatility of policy shock volatility	σ_e	3.0665×10^{-6}

suggest to the asymmetric Taylor rules that the U.S. does not react to the depreciation rate while the U.K. does.

3.4.2 Simulated Moments of the Model

The purpose of this work is to show the ability of the model accounting for a set of sample moments of exchange rate and interest rates in replicating as much as possible of the actual data. Ultimately, the model should produce a negative UIP coefficient and currency risk premium. Table (3.4) reports actual data and sample moments of the real economy. In terms of the real variables, the model replicates the actual data fairly well. The mean and variance of consumption growth, real interest rate and real depreciation of the model are reasonably close to the actual data. The autocorrelation of consumption is quite low. The cross country correlation in consumption growth

Table 3.2: Taylor Rules Parameters and Matching Moments

Taylor Rules	Parameter	Matching Moment: A	Matching Moment: B
<i>Basic Rule</i>			
$i_t = \tau + \tau_\pi \pi_t + \tau_z z_t + v$	τ, τ^* ,	$E(\pi_t), E(\pi_t^*),$	$var(\Delta s_{t+1}), \beta_{UIP},$
$i_t^* = \tau^* + \tau_\pi^* \pi_t^* + \tau_z^* z_t^* + v^*$	$\tau_\pi, \tau_\pi^*, \tau_z, \tau_z^*$	$var(\pi_t), var(\pi_t^*),$ $var(i_t), var(i_t^*)$	$E(\pi_t), E(\pi_t^*),$ $var(\pi_t), var(\pi_t^*)$
<i>Interest Rate Smoothing</i>			
$i_t = \tau + \tau_\pi \pi_t + \tau_z z_t + \tau_i i_{t-1} + v$	$\tau, \tau^*, \tau_\pi, \tau_\pi^*,$	$E(\pi_t), E(\pi_t^*),$	$var(\Delta s_{t+1}), \beta_{UIP},$
$i_t^* = \tau^* + \tau_\pi^* \pi_t^* + \tau_z^* z_t^* + \tau_i^* i_{t-1}^* + v^*$	$\tau_z, \tau_z^*, \tau_i, \tau_i^*$	$var(\pi_t), var(\pi_t^*),$ $var(i_t), var(i_t^*),$ $cov(i_t, \pi_t), cov(i_t^*, \pi_t^*)$	$E(\pi_t), E(\pi_t^*),$ $var(\pi_t), var(\pi_t^*),$ $var(i_t), var(i_t^*)$
<i>With Exchange Rate</i>			
$i_t = \tau + \tau_\pi \pi_t + \tau_z z_t + \tau_s \Delta s_t + v$	$\tau, \tau^*, \tau_\pi, \tau_\pi^*,$	$E(\pi_t), E(\pi_t^*),$	$var(\Delta s_{t+1}), \beta_{UIP},$
$i_t^* = \tau^* + \tau_\pi^* \pi_t^* + \tau_z^* z_t^* + \tau_s^* \Delta s_t^* + v^*$	$\tau_z, \tau_z^*, \tau_s, \tau_s^*$	$var(\pi_t), var(\pi_t^*),$ $var(i_t), var(i_t^*),$ $cov(i_t, i_t^*), cov(\pi_t, \pi_t^*)$	$E(\pi_t), E(\pi_t^*),$ $var(\pi_t), var(\pi_t^*),$ $var(i_t), var(i_t^*)$

is even lower than in the data. The calibration of the model emphasizes highly autocorrelated of real interest rate, and also highly cross-country correlation. The real UIP coefficient is negative as in the data, but the magnitude is much bigger.

Table (3.5), (3.6), and (3.7) display actual data and sample moments of the nominal variables, namely, inflation, nominal interest rate, nominal depreciation and UIP coefficient, according to the type of Taylor rules: basic rule, interest rate smoothing, and rule with exchange rate, respectively. The model from the three rules perform quite well to replicate the actual data of inflation and nominal interest rate, when matching with inflation-interest rate relation (Model A). The sample moments of inflation and the sample moment of nominal interest rate are pretty closed to the data. In terms of nominal exchange rate, the models deliver almost exact mean and variance of nominal depreciation rate as in the data.

Regarding UIP coefficient, it would be best to have a model that delivers a negative UIP coefficient. Unfortunately, the result from the three models with matching method A cannot deliver a negative UIP coefficient. However, these models produce UIP coefficients that are less than one, suggesting a deviation of UIP. Turning the model to match Taylor rule parameters with exchange rate volatility and UIP coefficient (Model B), the models perform well to deliver a negative UIP

Table 3.3: Value of Calibrated Nominal Parameters

Parameter	Basic Rule		Interest Rate Lag Rule		Exchange Rate Rule	
	Match A	Match B	Match A	Match B	Match A	Match B
τ	-0.0037	-0.1935	-0.0163	-0.2019	-0.0039	-0.2114
τ_π	1.8482	16.3479	1.2487	16.1551	1.9129	17.7126
τ_z	2.2919	1.0359	0.1835	1.8661	1.5294×10^{-4}	2.4914
τ_i			0.9999	0.5322		
τ_s					0.4750	0.0161
τ^*	-0.0056	-0.1413	-0.0065	-0.1504	-0.0057	-0.1460
τ_π^*	2.3210	16.1640	2.4065	15.6278	2.4517	16.6402
τ_z^*	0.9403	1.3655	0.0002	1.9801	4.4855×10^{-5}	2.8193
τ_i^*			0.0009	0.8353		
τ_s^*					0.1536	1.6060×10^{-6}

coefficient, which is about -0.0782 . However, the models still struggle to replicate a risk premium on GBP currency. The data has shown the excess return on GBP currency about 0.0040. This amount is positive but quite small. Although the risk premium generated by the model is positive, its amount is minuscule and far less than that of the data.

In short, some of the model's key moments are very different from observed moments. For the real variables calibration, the model moments of the autocorrelation of consumption growth and the autocorrelation of real interest rate are quite different from the actual data because I do not perform the calibration to match with these autocorrelations. It is difficult that the model will produce the (unmatched) moments close to the data. For the nominal variables calibration, there are two calibration procedures. The first way targets to match the model with domestic and foreign inflation-interest rate (Model A). It turns out that the model results in the moments of inflation, and interest rate that are very close to the actual data, but it cannot produce a negative UIP coefficient. The model has a limitation to match UIP coefficient with the actual data in the sense of matching the model with inflation-interest rate relation. The second method targets to match the model with the variance of nominal exchange rate depreciation and UIP coefficient (Model B). Although the inflation and the interest rate moments are not close to the actual data as well as in

Table 3.4: Real Variables

Moment	Data	Model
Consumption Growth (US)		
Mean	0.0074	0.0074
Variance	4.1763×10^{-5}	3.6458×10^{-5}
Autocorrelation	0.3323	0.0052
Cross-Country Correlation	0.5453	0.1852
Real Interest Rate (UK)		
Mean	0.0074	0.0074
Variance	9.520×10^{-5}	1.7238×10^{-7}
Autocorrelation	0.4524	0.9699
Cross-Country Correlation	0.5858	0.9961
Real Depreciation Rate		
Mean	-1.6684×10^{-4}	-3.9935×10^{-6}
Variance	0.0020	0.0020
Real UIP Coefficient	-0.1479	-11.5984

Model A, it appears that the model can produce a negative UIP coefficient, which is the objective in the study. However, the model is still unable to account for a sizable risk premium. This is a limitation of the model.

3.4.3 Responses to Shocks

In this section, I study dynamic responses of the main variables in the model: exchange rate depreciation rate, risk premium, consumption growth, inflation and interest rate to the shocks. This analysis is performed based on parameter values derived from matching the model with the variance of nominal exchange rate depreciation and UIP coefficient. I consider the impact of short-run consumption shock, long-run shock, and monetary policy shock both in level shock and volatility shock. The impulse responses are shown in figures (3.1) through (3.6).

Nominal Depreciation Rate: According to the impulse-response analysis, an increase in both short-run consumption growth level shock (ε_t^x) and long-run consumption level shock (ε_t^z) induce the exchange rate to depreciate. However, the exchange rate would appreciate in response to a rise

Table 3.5: Nominal Variables (Basic Rule)

Moment	Data	Model A	Model B
Inflation			
<i>Domestic: U.K.</i>			
Mean	0.0131	0.0131	0.0131
Variance	2.1520×10^{-4}	1.9862×10^{-4}	8.8808×10^{-7}
Autocorrelation	0.6121	0.7881	0.7868
<i>Foreign: U.S.</i>			
Mean	0.0098	0.0098	0.0098
Variance	7.3363×10^{-5}	9.3116×10^{-5}	9.0747×10^{-7}
Autocorrelation	0.6602	0.7868	0.7869
Nominal Interest Rate			
<i>Domestic: U.K.</i>			
Mean	0.0138	0.0204	0.0205
Variance	1.0294×10^{-4}	1.2850×10^{-4}	7.3607×10^{-7}
Autocorrelation	0.9876	0.7877	0.8231
<i>Foreign: U.S.</i>			
Mean	0.0135	0.0172	0.0172
Variance	9.0930×10^{-5}	6.0465×10^{-5}	7.3793×10^{-7}
Autocorrelation	0.9676	0.7870	0.8209
Nominal Depreciation Rate and UIP			
Mean	0.0031	0.0033	0.0033
Variance	0.0020	0.0020	0.0020
Autocorrelation	0.2650	0.0044	-0.0051
UIP Coefficient	-0.0782	0.9711	-0.0782
Risk Premium on GBP	0.0040	1.0750×10^{-5}	5.7017×10^{-6}

in short-run consumption volatility shock (ε_t^u), and a long-run volatility shock (ε_t^w). A decrease in the consumption growth of the economy and an increase in volatility component would lead to a reduce in consumption equilibrium and asset prices. The exchange rate as a price of the foreign currency drops, leading to an appreciation of the home currency. The impact of the short-run shock dies at time $t = 2$ onward. But the impact of the long-run shock is very persistent. The depreciation rates do not go to exactly zero for a long period of time. In terms of the monetary policy shock, an increase in the monetary policy level shock (ε_t^v) and the policy volatility shock (ε_t^e) immediately appreciates the exchange rate, then gradually depreciates the exchange rate.

Nominal Currency Risk Premium: Given the value of parameters generated in the model, a change in both short-run and long-run consumption growth level shocks has no impact on the excess return on foreign deposits. A shock in consumption growth impacts the depreciation rate while the interest rate differential adjusts according to the Taylor rule. The effect of shocks to change in the depreciation rate is cancelling out with the change in the interest rate differential. Nevertheless, increases in both short- and long-run consumption growth volatility shocks contribute to a rise in these excess returns. This is because a rise in uncertainty in the economy, as a result of an increase in volatility shocks, requires a positive excess return for an agent to hold risky assets. This supports the finding from Lustig and Verdelhan (2007) that higher foreign interest rates predict higher excess returns for a domestic investor in the foreign currency markets, and these excess returns compensate the domestic investor for taking on more domestic consumption growth risk. Besides, the currency risk premium also increases in response to an increase in the monetary-policy volatility shock. This is consistent with Eichenbaum and Evan (1995) that an increase in monetary policy shock produces an appreciation of the exchange rate and a deviation from UIP in the form of positive currency risk premium.

Consumption Growth: Consumption growth rate is only altered by short- and long-run consumption shocks. Other shocks have no impact on consumption growth at all. An increase in a short-run consumption shock leads to an increase in consumption growth. The effect is short and down to zero from period $t = 2$ onward. An increase in long-run consumption shock also contribute to a rise in consumption growth rate, and then the consumption growth gradually diminishes. The difference between these two shocks is a time period that the shock would affect. As by its name, the short-run consumption shock only hits the consumption growth to grow for only one period and then the effect dies down. However, the long-run shock has very persistent impact on the consumption growth.

Inflation: Response of inflation to a change in short- and long-run both level and volatility shocks is ambiguous. The consequence also depends on what type of the Taylor rules model relied on. An increase in short- and long-run consumption growth level shock causes inflation rate to

fall. The magnitude of the reduction varies according to the specification of the policy rule. For example, in response to a long-run consumption growth level shock, ε_t^z , the inflation derived from the exchange rate rule has a largest drop because ε_t^z has both direct impact on inflation itself, and indirect impact through the consumption processes. So the effect of ε_t^z on inflation is larger than the other two rules.

An increase in short- and long-run consumption volatility shock lowers inflation rate from the model derived from the basic rule and interest rate smoothing rule. But in the model with exchange rate rule, inflation increases in the first period of the shock and later inflation decreases.

Nominal Interest Rate: Regarding the nominal interest rate, fluctuations in monetary-policy level shock (ε_t^v) and also a monetary-policy volatility shock (ε_t^e) have both direct and indirect effects on the interest rate. An increase in policy level shock and policy volatility shock directly induce a rise in the interest rate. These shocks also have an indirect effect through the inflation. Policy shocks could alter the inflation which indirectly have an impact on the interest rate. This effect possibly causes a decrease in the interest rate. The effect of policy shocks on interest rate weighs on these forces.

A short-run consumption shock has no impact on the nominal interest rate while a long-run shock induces an increase in the interest rate. On the contrary, the interest rate declines in response to a rise in short- and long-run volatility shocks. This is due to a set-up of the model that the Taylor rule does not depend on the consumption growth but the long-run component in expected consumption growth. The implied inflation and hence the interest rate is a function of the long-run consumption shock, short- and long-run volatility shocks.

3.5 Conclusion

In this chapter, I study the behavior of exchange rate and currency risk premium in the context of long-run risk model. The idea is to model consumption growth containing a small and persistent component with stochastic volatility. I show that the presence of stochastic volatility of consumption growth influence the UIP violation.

The idea of this study is initiated from Backus et al. (2010) that remark the difficulties of the long-run risk model incorporating monetary policy through Taylor rule to simultaneously account for a negative UIP coefficient and a sizeable currency risk premium. The adjustment from their work is that I allow for a richer specification of the Taylor rule and expect the model to do better in explaining deviation from UIP. My model replicates the actual data fairly well. However, it is still difficult to match nominal variables with actual data in the sense of inflation-interest rate relation and delivers a negative UIP coefficient. Only I have to target to match these nominal variables with UIP coefficient and volatility of exchange rate depreciation rate, then the model could deliver a negative UIP coefficient. But the model still has a problem to cope with a currency risk premium. This is a limitation of the model.

I study how risks in the model affect the exchange rate and currency risk premium. I find that a decrease in short- and long-run consumption growth level shocks, and an increase in short- and long-run consumption growth volatility shocks lead to an appreciation of exchange rate. And, an increase in consumption growth volatility shock raises the excess return on foreign currency.

3.6 References

- Alvarez, Fernando; Atkeson, Andrew and Kehoe, Patrick J. "Time-Varying Risk, Interest Rates, and Exchange Rates in General Equilibrium." *The Review of Economics Studies*, 2009, 76(3), pp. 851-878.
- Alvarez, Fernando and Jermann, Urban J. "Quantitative Asset Pricing Implications of Endogenous Solvency Constraint." *The Review of Financial Studies*, 2001, 14(40), pp. 1117-1151.
- Backus, David K.; Foresi, Silverio and Telmer, Chris I. "Affine Term Structure Models and the Forward Premium Anomaly." *Journal of Finance*, 2001, 56(1), pp.279-304.
- Backus, David K.; Gavazzoni, Federico; Telmer, Chris and Zin, Stanley E. "Monetary Policy and the Uncovered Interest Rate Parity Puzzle." NBER Working Paper, 2010.
- Backus, David K. and Smith, Gregor W. "Consumption and Real Exchange Rates in Dynamic Economies with Non-Traded Goods." *Journal of International Economics*, 1993, 35(34), pp. 297-316.
- Ball, Laurence. "Policy Rules for Open Economies," in John B. Taylor, ed., *Monetary Policy Rules*. Chicago: University of Chicago Press, 1999, pp. 127-144.

- Bansal, Ravi; Gallant, Ronald and Tauchen, George. "Rational Pessimism, Rational Exuberance, and Asset Pricing Models." *The Review of Economic Studies*, 2007, 74(4), pp. 1005-1033.
- Bansal, Ravi; Khatchatrian, Varoujan and Yaron, Amir. "Interpretable asset markets?" *European Economic Review*, 2005, 49(3), pp. 531-560.
- Bansal, Ravi; Kiku, Dana and Yaron, Amir. "An Empirical Evaluation of the Long-Run Risks Model for Asset Prices." *Critical Finance Review*, 2012, 1, pp.183-221.
- Bansal, Ravi and Shaliastovich. "A Long-Run Risks Explanation of Predictability Puzzles in Bond and Currency Markets." *The Review of Financial Studies*, 2013, 26(1), pp. 1-33.
- Bansal, Ravi and Yaron, Amir. "Risks for the Long Run: A Potential Resolution of Asset Pricing Puzzles." *Journal of Finance*, 2004, 59, pp. 1481-1509.
- Batini, Nicoletta; Harrison, Richard and Millard, Stephen P. "Monetary Policy Rules for an Open Economy." *Journal of Economic Dynamics and Control*, 2003, 27(11-12), pp. 2059-2094.
- Benigno, Gianluca and Benigno, Pierpaolo. "Monetary Policy Rules and the Exchange Rate." Discussion Paper No. 2807, Center for Economic Policy Research, 2001
- Benigno, Gianluca; Nisticò, Salvatore and Benigno, Pierpaolo. "Risk, Monetary Policy, and the Exchange Rate." *NBER Macroeconomics Annual*, 2012, 26(1), pp. 247-309.
- Calvo, Guillermo A. "Staggered Prices in a Utility-Maximizing Framework." *Journal of Monetary Economics*, 1983, 12(3), pp. 383-398.
- Colacito, Riccardo and Croce, Mariano M. "Risks for the Long Run and the Real Exchange Rate." *Journal of Political Economy*, 2011, 119(1), pp. 153-181.
- Colacito, Riccardo and Croce, Mariano M. "International Asset Pricing with Recursive Preferences." *Journal of Finance*, 2013, 68(6), pp. 2651-2685.
- Colacito, Riccardo; Croce, Mariano M.; Gavazzoni, Federico and Ready, Robert C. "Currency Risk Factors in a Recursive Multi-Country Economy." *Journal of Finance*, forthcoming, 2017.
- Eichenbaum, Martin and Evans, Charles L. "Some Empirical Evidence on the Effects of Shocks to Monetary Policy on Exchange Rates." *The Quarterly Journal of Economics*, 1995, 110(4), pp. 975-1009.
- Epstein, Larry G and Zin, Stanley E. "Substitution, Risk Aversion, and the Temporal Behavior of Consumption and Asset Returns: A Theoretical Framework." *Econometrica*, 1989, 57(4), pp. 937-969.
- Fama, Eugene F. "Forward and Spot Exchange Rates." *Journal of Monetary Economics*, 1984, 14(3), pp. 319-338.
- Froyen, Richard T. and Guender, Alfred V. "The Real Exchange Rate in Open-Economy Taylor Rules: A Re-Assessment." Working Paper, Economic 16/10, University of Canterbury, 2016

- Hansen, Lars Peter; Heaton, John C. and Li, Nan. "Consumption Strikes Back? Measuring Long-Run Risk." *Journal of Political Economy*, 2008, 116(2), pp. 260-302.
- Hansen, Lars Peter and Sargent, Thomas. "Fragile Beliefs and the Price of Model Uncertainty." Unpublished manuscript, 2007.
- Leitemo, Kai and Söderström, Ulf. "Simple Monetary Policy Rules and Exchange Rate Uncertainty." *Journal of International Money and Finance*, 2005, 24, pp. 481-507.
- Lustig, Hanno and Verdelhan, Adrien. "The Cross Section of Foreign Currency Risk Premia and Consumption Growth Risk." *American Economic Review*, 2007, 97(1), pp. 89-117.
- Mehra, Rajnish and Prescott, Edward C. "The Equity Premium: A Puzzle." *Journal of Monetary Economics*, 1985, 15(2), pp. 145-161.
- Shiller, Robert J. "Do Stock Prices Move Too Much to Be Justified by Subsequent Changes in dividends?" *American Economic Review*, 1981, 71, pp. 421-436.
- Svensson, Lars E.O. "Open-Economy Inflation Targeting." *Journal of International Economics*, 2000, 50(1), pp. 155-183.
- Taylor, John B. "The Role of the Exchange Rate in Monetary-Policy Rules." *American Economic Review*, 2001, 91(2), pp. 263-267.
- Verdelhan, Adrien. "A Habit-Based Explanation of the Exchange Rate Risk Premium." *Journal of Finance*, 2010, 65(1), pp. 123-146.

Table 3.6: Nominal Variables (Smoothing Interest Rate Rule)

Moment	Data	Model A	Model B
Inflation			
<i>Domestic: U.K.</i>			
Mean	0.0131	0.0131	0.0131
Variance	2.1520×10^{-4}	1.8646×10^{-4}	8.7455×10^{-7}
Autocorrelation	0.6121	0.5507	0.7776
<i>Foreign: U.S.</i>			
Mean	0.0098	0.0098	0.0098
Variance	7.3363×10^{-5}	8.3431×10^{-5}	9.1235×10^{-7}
Autocorrelation	0.6602	0.7868	0.7717
Nominal Interest Rate			
<i>Domestic: U.K.</i>			
Mean	0.0138	0.0204	0.0205
Variance	1.0294×10^{-4}	6.6878×10^{-5}	6.7689×10^{-7}
Autocorrelation	0.9876	0.7870	0.8179
Covariance (i_t, π_t)	2.7447×10^{-5}	1.0674×10^{-4}	
<i>Foreign: U.S.</i>			
Mean	0.0135	0.0172	0.0172
Variance	9.0930×10^{-5}	5.4538×10^{-5}	6.8203×10^{-7}
Autocorrelation	0.9676	0.7883	0.8156
Covariance (i_t^*, π_t^*)	5.0440×10^{-5}	6.7317×10^{-5}	
Nominal Depreciation Rate and UIP			
Mean	0.0031	0.0032	0.0033
Variance	0.0020	0.0021	0.0020
Autocorrelation	0.2605	-0.0024	-0.0051
UIP Coefficient	-0.0782	0.9196	-0.0782
Risk Premium on GBP	0.0040	-3.6544×10^{-5}	5.8553×10^{-6}

Table 3.7: Nominal Variables (Exchange Rate Rule)

Moment	Data	Model A	Model B
Inflation			
<i>Domestic: U.K.</i>			
Mean	0.0131	0.0131	0.0131
Variance	2.1520×10^{-4}	2.1612×10^{-4}	7.5671×10^{-7}
Autocorrelation	0.6121	0.5165	0.7861
Covariance (π_t, π_t^*)	0.6603	0.6469	0.9911
<i>Foreign: U.S.</i>			
Mean	0.0098	0.0098	0.0098
Variance	7.3363×10^{-5}	8.8179×10^{-5}	8.6024×10^{-7}
Autocorrelation	0.6602	0.7432	0.7883
Nominal Interest Rate			
<i>Domestic: U.K.</i>			
Mean	0.0138	0.0227	0.0205
Variance	1.0294×10^{-4}	9.3687×10^{-5}	6.0780×10^{-7}
Autocorrelation	0.9876	0.7882	0.8202
Covariance (i_t, i_t^*)	0.8944	0.9959	0.9910
<i>Foreign: U.S.</i>			
Mean	0.0135	0.0178	0.0172
Variance	9.0930×10^{-5}	5.4659×10^{-5}	6.6287×10^{-7}
Autocorrelation	0.9676	0.7884	0.8148
Nominal Depreciation Rate and UIP			
Mean	0.0031	0.0033	0.0033
Variance	0.0020	0.0012	0.0020
Autocorrelation	0.2605	0.0012	-0.0051
UIP Coefficient	-0.0782	0.7457	-0.0782
Risk Premium on GBP	0.0040	0.0016	1.9217×10^{-5}

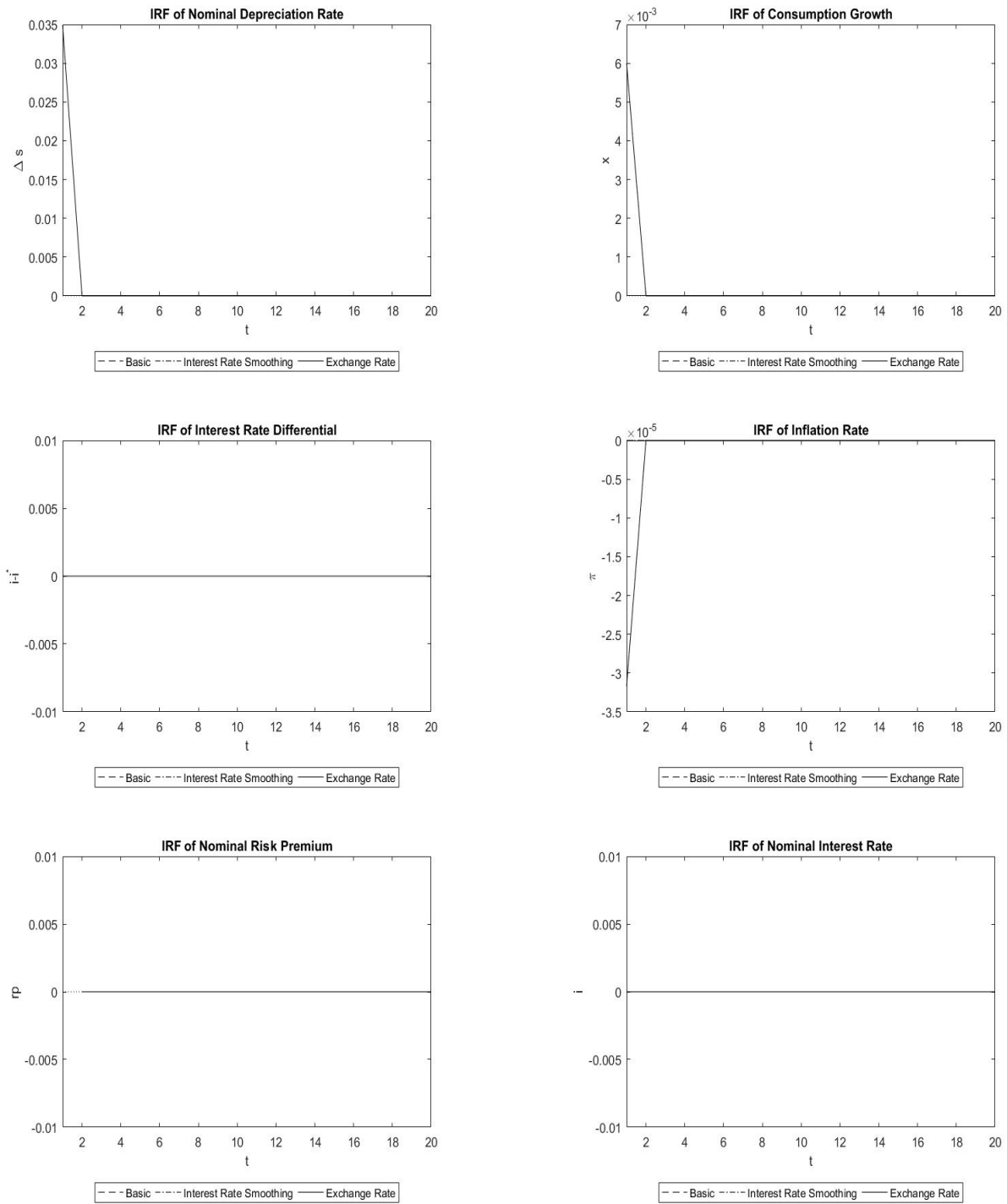


Figure 3.1: Dynamic Responses to the Short-Run Consumption Level Shock, ε_t^x

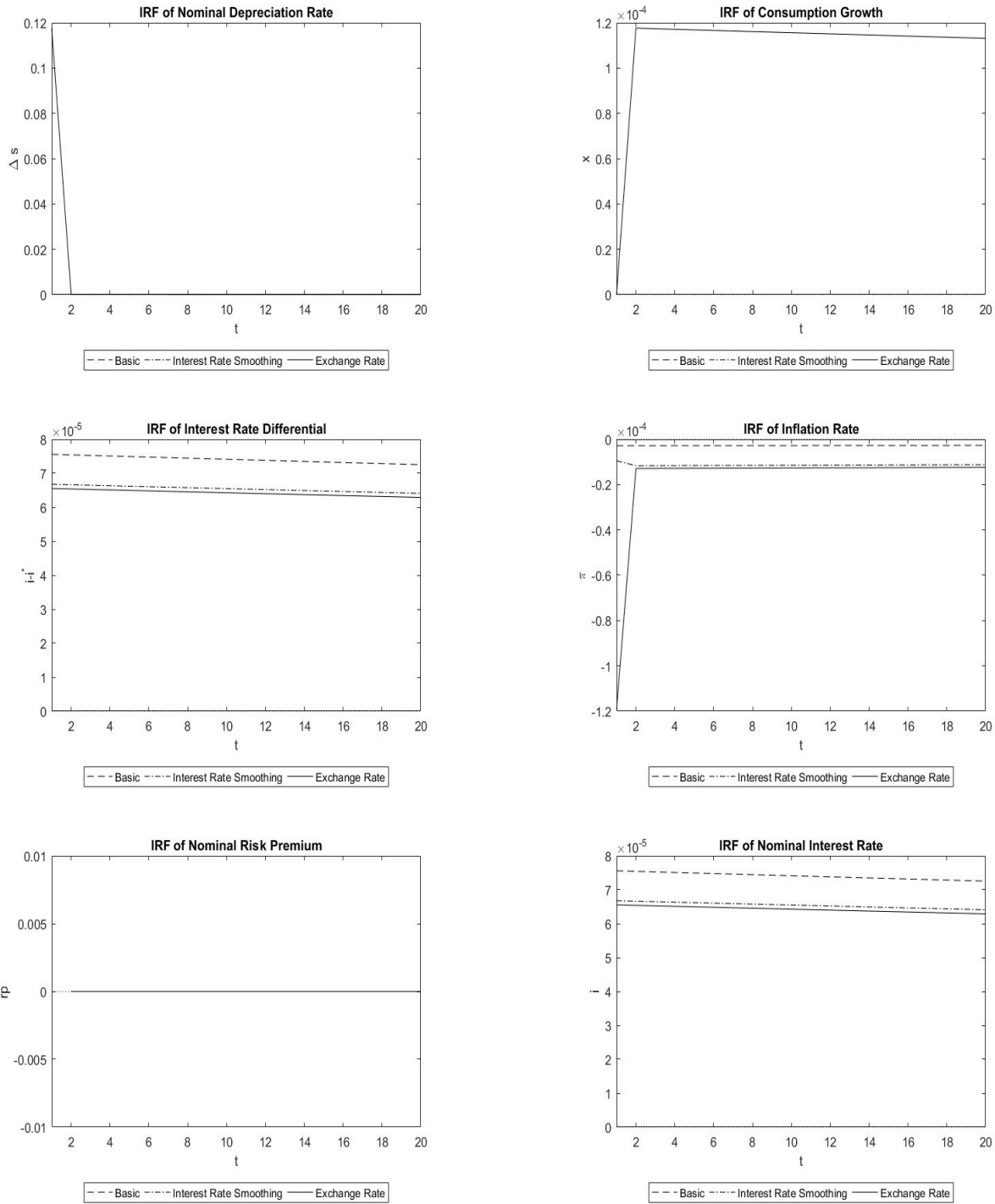


Figure 3.2: Dynamic Responses to the Long-Run Consumption Level Shock, ε_t^z

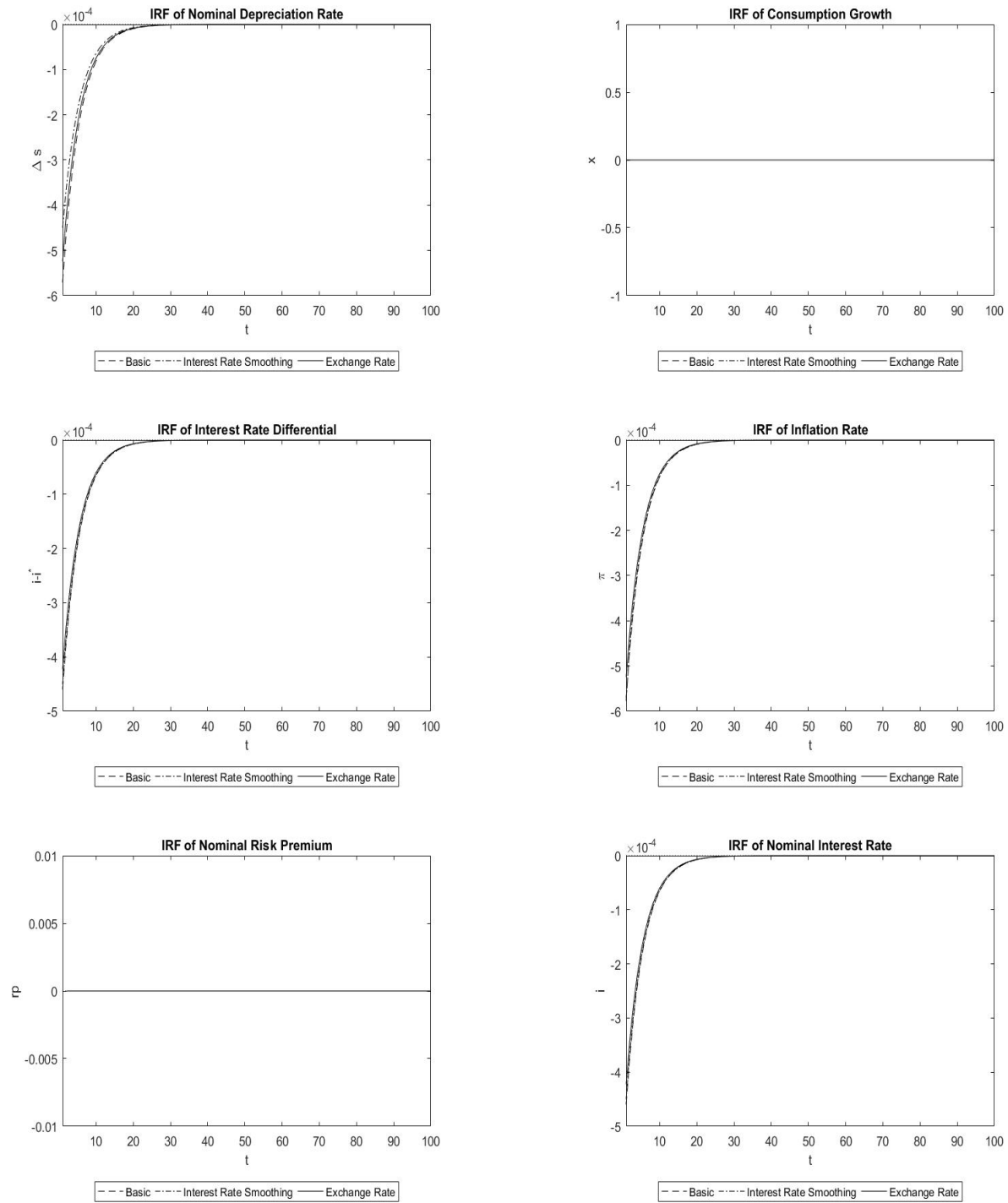


Figure 3.3: Dynamic Responses to the Monetary Policy Level Shock, ε_t^v

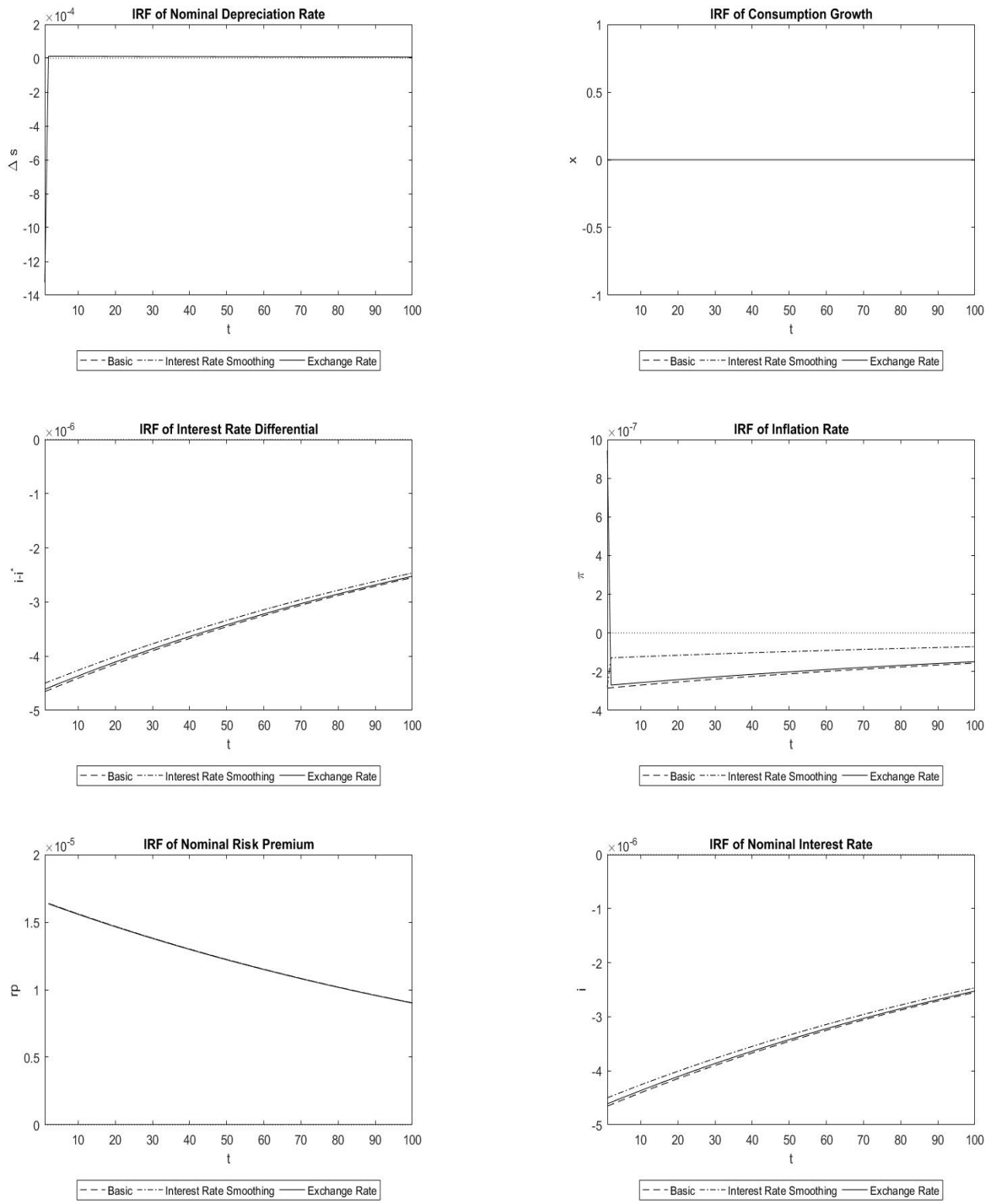


Figure 3.4: Dynamic Responses to the Short-Run Consumption Volatility Shock, ε_t^u

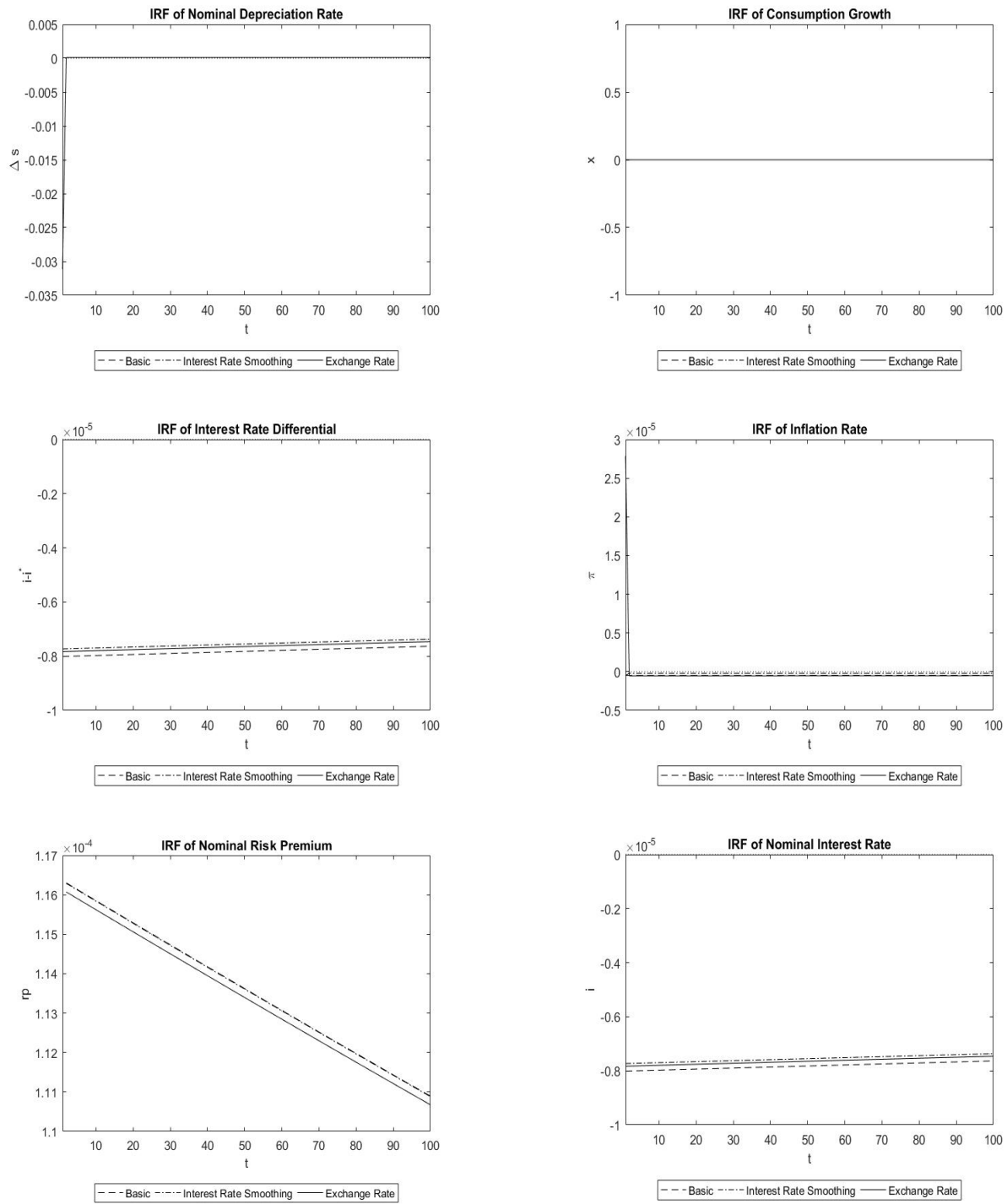


Figure 3.5: Dynamic Responses to the Long-Run Consumption Volatility Shock, ε_t^w

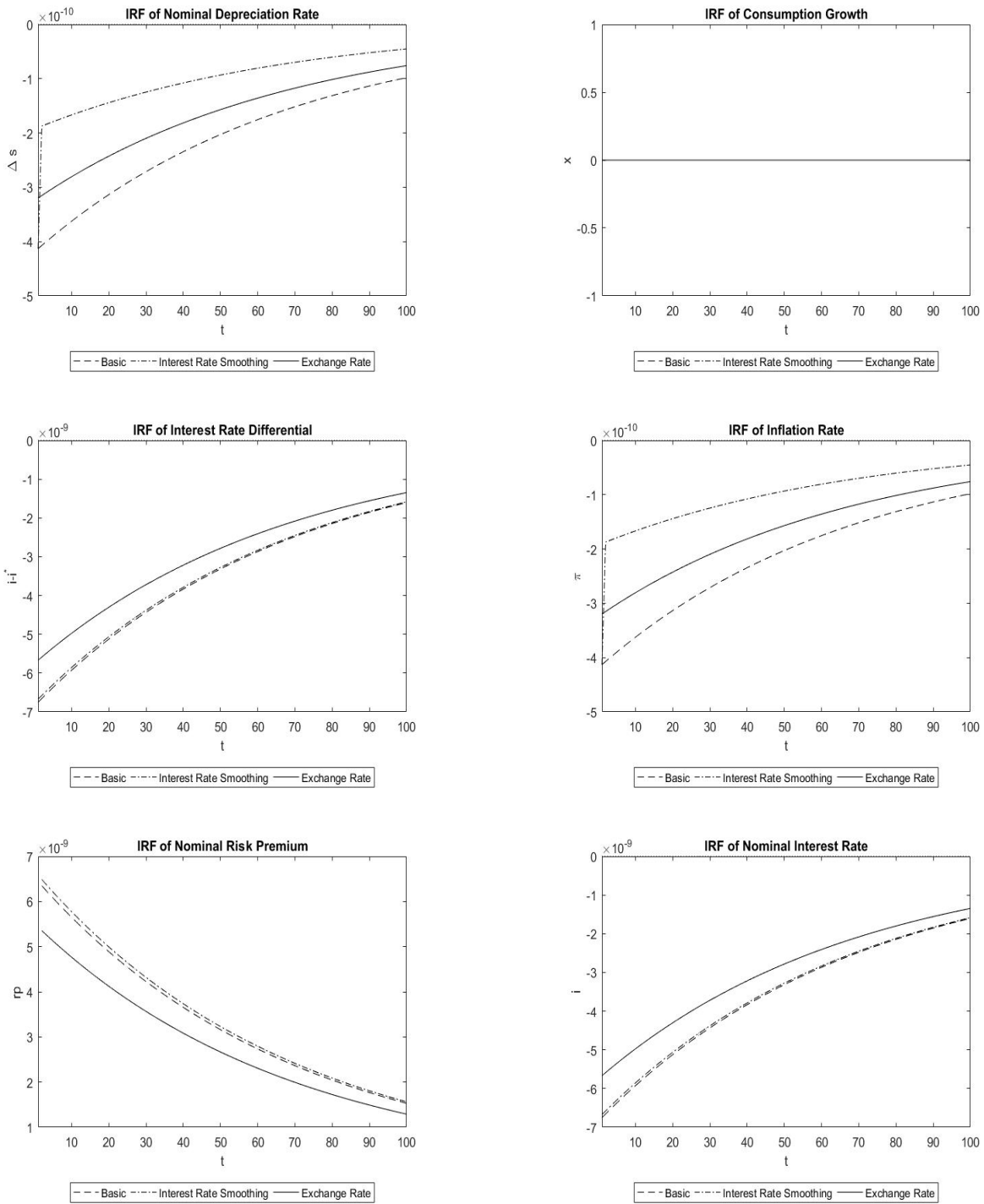


Figure 3.6: Dynamic Responses to the Monetary Policy Volatility Shock, ε_t^e

APPENDIX A. CHAPTER 1 APPENDIX

A.1 Higher-Order Iterated Expectation

Allen et al. (2006) explain this situation by considering the case where there is no learning over time. Suppose θ is distributed normally with mean y and variance $\frac{1}{\alpha}$. Each agent i in a continuum observes a signal $x_i = \theta + \varepsilon_i$, where ε_i is distributed in the population with mean 0 and variance $\frac{1}{\beta}$. Suppose that this is all the information available at all dates. Then we may drop the date subscripts.

Since $\theta \sim N(y, \frac{1}{\alpha})$ and $x_i|\theta \sim N(\theta, \frac{1}{\beta})$, so

$$\begin{aligned} f(\theta, x_i) = f(x_i|\theta)f_\theta(\theta) &= \frac{1}{\sqrt{2\pi/\beta}} \exp\left(-\frac{1}{2/\beta}(x_i - \theta)^2\right) \frac{1}{\sqrt{2\pi/\alpha}} \exp\left(-\frac{1}{2/\alpha}(\theta - y)^2\right) \\ &= \frac{1}{2\pi/\sqrt{\alpha\beta}} \exp\left(-\frac{1}{2}(\beta(x_i - \theta)^2 + \alpha(\theta - y)^2)\right). \end{aligned}$$

Since $x_i \sim N(y, \frac{1}{\alpha} + \frac{1}{\beta})$, hence

$$\begin{aligned} f(\theta|x_i) &= \frac{f(\theta, x_i)}{f_{x_i}(x_i)} = \frac{\frac{1}{2\pi/\sqrt{\alpha\beta}} \exp\left(-\frac{1}{2}(\beta(x_i - \theta)^2 + \alpha(\theta - y)^2)\right)}{\frac{1}{\sqrt{2\pi(\frac{1}{\alpha} + \frac{1}{\beta})}} \exp\left(-\frac{1}{2(\frac{1}{\alpha} + \frac{1}{\beta})}(x_i - y)^2\right)} \\ &= \frac{1}{\sqrt{2\pi/(\alpha + \beta)}} \exp\left(-\frac{1}{2}\left(\beta(x_i - \theta)^2 + \alpha(\theta - y)^2 - \frac{\alpha\beta}{\alpha + \beta}(x_i - y)^2\right)\right) \\ &= \frac{1}{\sqrt{2\pi/(\alpha + \beta)}} \exp\left(-\frac{1}{2}\left(\frac{(\alpha + \beta)^2\theta^2 - 2(\alpha + \beta)(\alpha y + \beta x_i)\theta + (\alpha y + \beta x_i)^2}{\alpha + \beta}\right)\right) \\ &= \frac{1}{\sqrt{2\pi/(\alpha + \beta)}} \exp\left(-\frac{1}{2}\left(\frac{[(\alpha + \beta)\theta - (\alpha y + \beta x_i)]^2}{\alpha + \beta}\right)\right) \\ &= \frac{1}{\sqrt{2\pi/(\alpha + \beta)}} \exp\left(-\frac{1}{2/(\alpha + \beta)}\left(\theta - \frac{\alpha y + \beta x_i}{\alpha + \beta}\right)^2\right) \end{aligned}$$

Therefore, $\theta|x_i \sim N\left(\frac{\alpha y + \beta x_i}{\alpha + \beta}, \frac{1}{\alpha + \beta}\right)$. One may rearrange the mean to be $\frac{\frac{1}{\beta}y + \frac{1}{\alpha}x}{\frac{1}{\alpha} + \frac{1}{\beta}}$ and variance to be

$\frac{\frac{1}{\alpha} \cdot \frac{1}{\beta}}{\frac{1}{\alpha} + \frac{1}{\beta}}$. Thus

$$\begin{aligned}
E^i[\theta] &= \frac{\alpha y + \beta x_i}{\alpha + \beta} \\
\bar{E}[\theta] &= \frac{\alpha y + \beta \theta}{\alpha + \beta} \\
E^i[\bar{E}[\theta]] &= \frac{\alpha y + \beta E^i[\theta]}{\alpha + \beta} \\
&= \frac{\alpha y + \beta \frac{\alpha y + \beta x_i}{\alpha + \beta}}{\alpha + \beta} \\
&= \left(1 - \left(\frac{\beta}{\alpha + \beta}\right)^2\right) y + \left(\frac{\beta}{\alpha + \beta}\right)^2 x_i \\
\bar{E}[\bar{E}[\theta]] &= \left(1 - \left(\frac{\beta}{\alpha + \beta}\right)^2\right) y + \left(\frac{\beta}{\alpha + \beta}\right)^2 \theta
\end{aligned}$$

Iterate this operation, one can show that

$$\bar{E}^k[\theta] = \left(1 - \left(\frac{\beta}{\alpha + \beta}\right)^k\right) y + \left(\frac{\beta}{\alpha + \beta}\right)^k \theta$$

Putting back the time subscripts, we have

$$\bar{E}_t[\bar{E}_{t+1}[\theta]] = \left(1 - \left(\frac{\beta}{\alpha + \beta}\right)^2\right) y + \left(\frac{\beta}{\alpha + \beta}\right)^2 \theta \neq \frac{\alpha y + \beta \theta}{\alpha + \beta} = \bar{E}_t[\theta]$$

and

$$\bar{E}_t[\bar{E}_{t+1}[\dots \bar{E}_{T-2}[\bar{E}_{T-1}[\theta]]]] = \left(1 - \left(\frac{\beta}{\alpha + \beta}\right)^{T-t}\right) y + \left(\frac{\beta}{\alpha + \beta}\right)^{T-t} \theta$$

A.2 Invertibility of b_t

From the conjectured solution $s_t = \lambda_0 f_t + \lambda_1 f_{t+1} + \lambda_2 f_{t+2} + \gamma_{-2} b_{t-2} + \gamma_{-1} b_{t-1} + \gamma_0 b_t$, I rearrange the equation to be $(\gamma_0 + \gamma_{-1}L + \gamma_{-2}L^2)b_t = s_t - \lambda_0 f_t - \lambda_1 f_{t+1} - \lambda_2 f_{t+2}$

Case 1: if $\gamma_{-2} = 0$, the process b_t is invertible if and only if root of polynomial $\gamma_0 + \gamma_{-1}z = 0$ lies outside unit circle. That is, $\left|\frac{\gamma_{-1}}{\gamma_0}\right| < 1$.

If b_t is invertible, then

$$\begin{aligned}
b_t &= \frac{1}{(\gamma_0 + \gamma_{-1}L)} [s_t - \lambda_0 f_t - \lambda_1 f_{t+1} - \lambda_2 f_{t+2}] \\
&= \frac{1}{\gamma_0 \left(1 + \frac{\gamma_{-1}L}{\gamma_0}\right)} [s_t - \lambda_0 f_t - \lambda_1 f_{t+1} - \lambda_2 f_{t+2}] \\
&= \frac{1 - \left(\frac{\gamma_{-1}}{\gamma_0}\right)L + \left(\frac{\gamma_{-1}}{\gamma_0}\right)^2 L^2 - \left(\frac{\gamma_{-1}}{\gamma_0}\right)^3 L^3 + \dots}{\gamma_0} [s_t - \lambda_0 f_t - \lambda_1 f_{t+1} - \lambda_2 f_{t+2}] \\
&= \frac{1}{\gamma_0} \left\{ \left(s_t - \left(\frac{\gamma_{-1}}{\gamma_0}\right) s_{t-1} + \left(\frac{\gamma_{-1}}{\gamma_0}\right)^2 s_{t-2} - \dots \right) \right. \\
&\quad - \lambda_0 \left(f_t - \left(\frac{\gamma_{-1}}{\gamma_0}\right) f_{t-1} + \left(\frac{\gamma_{-1}}{\gamma_0}\right)^2 f_{t-2} - \dots \right) \\
&\quad - \lambda_1 \left(f_{t+1} - \left(\frac{\gamma_{-1}}{\gamma_0}\right) f_t + \left(\frac{\gamma_{-1}}{\gamma_0}\right)^2 f_{t-1} - \dots \right) \\
&\quad \left. - \lambda_2 \left(f_{t+2} - \left(\frac{\gamma_{-1}}{\gamma_0}\right) f_{t+1} + \left(\frac{\gamma_{-1}}{\gamma_0}\right)^2 f_t - \dots \right) \right\}.
\end{aligned}$$

Case 2: if $\gamma_{-2} \neq 0$, the process b_t is invertible if and only if root of polynomial $\gamma_0 + \gamma_{-1}z + \gamma_{-2}z^2 = 0$ lies outside unit circle. That is, $|z_1| = \left| \frac{\gamma_{-1} + \sqrt{\gamma_{-1}^2 - 4\gamma_0\gamma_{-2}}}{2\gamma_{-2}} \right| > 1$ and $|z_2| = \left| \frac{\gamma_{-1} - \sqrt{\gamma_{-1}^2 - 4\gamma_0\gamma_{-2}}}{2\gamma_{-2}} \right| > 1$.

If b_t is invertible, then

$$\begin{aligned}
b_t &= \frac{1}{(\gamma_0 + \gamma_{-1}L + \gamma_{-2}L^2)} [s_t - \lambda_0 f_t - \lambda_1 f_{t+1} - \lambda_2 f_{t+2}] \\
&= \frac{1}{\gamma_0 \left(1 + \frac{\gamma_{-1}L}{\gamma_0} + \frac{\gamma_{-2}L^2}{\gamma_0}\right)} [s_t - \lambda_0 f_t - \lambda_1 f_{t+1} - \lambda_2 f_{t+2}] \\
&= \frac{1}{\gamma_0 \left(1 - \frac{1}{z_1}L\right) \left(1 - \frac{1}{z_2}L\right)} [s_t - \lambda_0 f_t - \lambda_1 f_{t+1} - \lambda_2 f_{t+2}] \\
&= \frac{1}{\gamma_0} \left\{ \left(1 + \frac{1}{z_1}L + \frac{1}{z_1^2}L^2 + \dots\right) \left(1 + \frac{1}{z_2}L + \frac{1}{z_2^2}L^2 + \dots\right) \right\} [s_t - \lambda_0 f_t - \lambda_1 f_{t+1} - \lambda_2 f_{t+2}] \\
&= \frac{1}{\gamma_0} \{1 + c_1L + c_2L^2 + \dots\} [s_t - \lambda_0 f_t - \lambda_1 f_{t+1} - \lambda_2 f_{t+2}] \\
&= \frac{1}{\gamma_0} \{ (s_t + c_1s_{t-1} + c_2s_{t-2} - \dots) - \lambda_0 (f_t + c_1f_{t-1} + c_2f_{t-2} - \dots) \\
&\quad - \lambda_1 (f_{t+1} + c_1f_t + c_2f_{t-1} - \dots) - \lambda_2 (f_{t+2} + c_1f_{t+1} + c_2f_t - \dots) \},
\end{aligned}$$

where $c_n = \sum_{i=0}^n \left(\frac{1}{z_1}\right)^i \left(\frac{1}{z_2}\right)^{n-i}$, $n = 1, 2, \dots$. At time t , b_t is unknown to the investor because b_t depends on future fundamental f_{t+1} and f_{t+2} . b_{t-1} is also unknown because it depends on

f_{t+1} . However, from b_{t-2} onward depends on current and past fundamentals and exchange rates. Therefore, every investor knows about b_{t-2} and the lags before.

A.3 Signal Extraction

A $k \times 1$ random vector of information $\boldsymbol{\theta} = \begin{pmatrix} \theta_1 & \dots & \theta_k \end{pmatrix}' \sim N(\boldsymbol{\mu}, \boldsymbol{\Sigma}_\theta)$, where $\boldsymbol{\mu}$ is a $k \times 1$ vector of means of $\boldsymbol{\theta}$ and $\boldsymbol{\Sigma}_\theta$ is a $k \times k$ symmetric positive definite variance-covariance matrix of $\boldsymbol{\theta}$, is transmitted through n channels. Each channel $i = 1, \dots, n$, there is a random noises ε_i which are independent of the information vector $\boldsymbol{\theta}$. Let $\boldsymbol{\varepsilon} = \begin{pmatrix} \varepsilon_1 & \dots & \varepsilon_n \end{pmatrix}'$ be an $n \times 1$ vector of random noises and assume that $\boldsymbol{\varepsilon} \sim N(\mathbf{0}, \boldsymbol{\Sigma}_\varepsilon)$, where $\mathbf{0}$ is an $n \times 1$ vector of zeros and $\boldsymbol{\Sigma}_\varepsilon$ is an $n \times n$ symmetric positive definite variance-covariance matrix of $\boldsymbol{\varepsilon}$. The recipient receives n signals, x_1, \dots, x_n . Let $\mathbf{x} = \begin{pmatrix} x_1 & \dots & x_n \end{pmatrix}'$ denote the $n \times 1$ vector of signals. The signals are generated by a linear process $\mathbf{x} = \mathbf{A}\boldsymbol{\theta} + \boldsymbol{\varepsilon}$, where \mathbf{A} is an $n \times k$ matrix with $\text{rank}(\mathbf{A}) = k$. Given the signal \mathbf{x} , the recipient wants to find out about the information $\boldsymbol{\theta}$.

Since $\mathbf{x} = \mathbf{A}\boldsymbol{\theta} + \boldsymbol{\varepsilon}$ and we know that $\boldsymbol{\varepsilon} \sim N(\mathbf{0}, \boldsymbol{\Sigma}_\varepsilon)$ and $\boldsymbol{\theta} \sim N(\boldsymbol{\mu}, \boldsymbol{\Sigma}_\theta)$, then $\mathbf{x}|\boldsymbol{\theta} \sim N(\mathbf{A}\boldsymbol{\theta}, \boldsymbol{\Sigma}_\varepsilon)$ and $\mathbf{x} \sim N(\mathbf{A}\boldsymbol{\mu}, \mathbf{A}\boldsymbol{\Sigma}_\theta\mathbf{A}' + \boldsymbol{\Sigma}_\varepsilon)$. The conditional pdf of $\boldsymbol{\theta}$ given \mathbf{x} is

$$\begin{aligned}
f(\boldsymbol{\theta}|\mathbf{x}) &= \frac{f(\boldsymbol{\theta}, \mathbf{x})}{f(\mathbf{x})} \\
&= \frac{f(\mathbf{x}|\boldsymbol{\theta})f(\boldsymbol{\theta})}{f(\mathbf{x})} \\
&= \frac{(2\pi)^{-\frac{n}{2}} |\boldsymbol{\Sigma}_\varepsilon|^{-\frac{1}{2}} \exp[-\frac{1}{2}(\mathbf{x} - \mathbf{A}\boldsymbol{\theta})' \boldsymbol{\Sigma}_\varepsilon^{-1} (\mathbf{x} - \mathbf{A}\boldsymbol{\theta})] (2\pi)^{-\frac{k}{2}} |\boldsymbol{\Sigma}_\theta|^{-\frac{1}{2}} \exp[-\frac{1}{2}(\boldsymbol{\theta} - \boldsymbol{\mu})' \boldsymbol{\Sigma}_\theta^{-1} (\boldsymbol{\theta} - \boldsymbol{\mu})]}{(2\pi)^{-\frac{n}{2}} |\mathbf{A}\boldsymbol{\Sigma}_\theta\mathbf{A}' + \boldsymbol{\Sigma}_\varepsilon|^{-\frac{1}{2}} \exp[-\frac{1}{2}(\mathbf{x} - \mathbf{A}\boldsymbol{\mu})' (\mathbf{A}\boldsymbol{\Sigma}_\theta\mathbf{A}' + \boldsymbol{\Sigma}_\varepsilon)^{-1} (\mathbf{x} - \mathbf{A}\boldsymbol{\mu})]} \\
&= \frac{(2\pi)^{-\frac{k}{2}} |\boldsymbol{\Sigma}_\varepsilon|^{-\frac{1}{2}} |\boldsymbol{\Sigma}_\theta|^{-\frac{1}{2}}}{|\mathbf{A}\boldsymbol{\Sigma}_\theta\mathbf{A}' + \boldsymbol{\Sigma}_\varepsilon|^{-\frac{1}{2}}} \exp \left[-\frac{1}{2} \left((\mathbf{x} - \mathbf{A}\boldsymbol{\theta})' \boldsymbol{\Sigma}_\varepsilon^{-1} (\mathbf{x} - \mathbf{A}\boldsymbol{\theta}) + (\boldsymbol{\theta} - \boldsymbol{\mu})' \boldsymbol{\Sigma}_\theta^{-1} (\boldsymbol{\theta} - \boldsymbol{\mu}) - (\mathbf{x} - \mathbf{A}\boldsymbol{\mu})' (\mathbf{A}\boldsymbol{\Sigma}_\theta\mathbf{A}' + \boldsymbol{\Sigma}_\varepsilon)^{-1} (\mathbf{x} - \mathbf{A}\boldsymbol{\mu}) \right) \right]
\end{aligned}$$

To simplify the notation, define $\mathbf{y} = \mathbf{x} - \mathbf{A}\boldsymbol{\mu}$, $\mathbf{z} = \boldsymbol{\theta} - \boldsymbol{\mu}$, and $\mathbf{B} = \boldsymbol{\Sigma}_\theta^{-1} + \mathbf{A}'\boldsymbol{\Sigma}_\varepsilon^{-1}\mathbf{A}$. Hence, $\mathbf{x} - \mathbf{A}\boldsymbol{\theta} = \mathbf{x} - \mathbf{A}\boldsymbol{\mu} + \mathbf{A}\boldsymbol{\mu} - \mathbf{A}\boldsymbol{\theta} = \mathbf{y} - \mathbf{A}\mathbf{z}$. Using Sherman-Morrison formula (one can see proof in appendix A (equation (A-66b)) from Greene (2012)) that

$$(\mathbf{K} \pm \mathbf{LML}'^{-1} = \mathbf{K}^{-1} \mp \mathbf{K}^{-1}\mathbf{L}(\mathbf{M}^{-1} \pm \mathbf{L}'\mathbf{K}^{-1}\mathbf{L})^{-1}\mathbf{L}'\mathbf{K}^{-1},$$

where \mathbf{K} and \mathbf{M} are symmetric invertible matrices and \mathbf{L} is a matrix, the term $(\mathbf{A}\Sigma_{\theta}\mathbf{A}' + \Sigma_{\varepsilon})^{-1}$ can be simplified as $\Sigma_{\varepsilon}^{-1} - \Sigma_{\varepsilon}^{-1}\mathbf{A}(\Sigma_{\theta}^{-1} + \mathbf{A}'\Sigma_{\varepsilon}^{-1}\mathbf{A})^{-1}\mathbf{A}'\Sigma_{\varepsilon}^{-1}$.

Rearranging the terms in the exponent.

$$\begin{aligned}
& (\mathbf{x} - \mathbf{A}\boldsymbol{\theta})'\Sigma_{\varepsilon}^{-1}(\mathbf{x} - \mathbf{A}\boldsymbol{\theta}) + (\boldsymbol{\theta} - \boldsymbol{\mu})'\Sigma_{\theta}^{-1}(\boldsymbol{\theta} - \boldsymbol{\mu}) - (\mathbf{x} - \mathbf{A}\boldsymbol{\mu})'(\mathbf{A}\Sigma_{\theta}\mathbf{A}' + \Sigma_{\varepsilon})^{-1}(\mathbf{x} - \mathbf{A}\boldsymbol{\mu}) \\
= & (\mathbf{y} - \mathbf{A}\mathbf{z})'\Sigma_{\varepsilon}^{-1}(\mathbf{y} - \mathbf{A}\mathbf{z}) + \mathbf{z}'\Sigma_{\theta}^{-1}\mathbf{z} - \mathbf{y}'(\Sigma_{\varepsilon}^{-1} - \Sigma_{\varepsilon}^{-1}\mathbf{A}(\Sigma_{\theta}^{-1} + \mathbf{A}'\Sigma_{\varepsilon}^{-1}\mathbf{A})^{-1}\mathbf{A}'\Sigma_{\varepsilon}^{-1})\mathbf{y} \\
= & \mathbf{y}'\Sigma_{\varepsilon}^{-1}\mathbf{y} - \mathbf{y}'\Sigma_{\varepsilon}^{-1}\mathbf{A}\mathbf{z} - \mathbf{z}'\mathbf{A}'\Sigma_{\varepsilon}^{-1}\mathbf{y} + \mathbf{z}'\mathbf{A}'\Sigma_{\varepsilon}^{-1}\mathbf{A}\mathbf{z} + \mathbf{z}'\Sigma_{\theta}^{-1}\mathbf{z} - \mathbf{y}'\Sigma_{\varepsilon}^{-1}\mathbf{y} \\
& + \mathbf{y}'\Sigma_{\varepsilon}^{-1}\mathbf{A}(\Sigma_{\theta}^{-1} + \mathbf{A}'\Sigma_{\varepsilon}^{-1}\mathbf{A})^{-1}\mathbf{A}'\Sigma_{\varepsilon}^{-1}\mathbf{y} \\
= & -\mathbf{y}'\Sigma_{\varepsilon}^{-1}\mathbf{A}\mathbf{z} - \mathbf{z}'\mathbf{A}'\Sigma_{\varepsilon}^{-1}\mathbf{y} + \mathbf{z}'(\mathbf{A}'\Sigma_{\varepsilon}^{-1}\mathbf{A} + \Sigma_{\theta}^{-1})\mathbf{z} + \mathbf{y}'\Sigma_{\varepsilon}^{-1}\mathbf{A}(\Sigma_{\theta}^{-1} + \mathbf{A}'\Sigma_{\varepsilon}^{-1}\mathbf{A})^{-1}\mathbf{A}'\Sigma_{\varepsilon}^{-1}\mathbf{y} \\
= & -\mathbf{y}'\Sigma_{\varepsilon}^{-1}\mathbf{A}\mathbf{z} - \mathbf{z}'\mathbf{A}'\Sigma_{\varepsilon}^{-1}\mathbf{y} + \mathbf{z}'\mathbf{B}\mathbf{z} + \mathbf{y}'\Sigma_{\varepsilon}^{-1}\mathbf{A}\mathbf{B}^{-1}\mathbf{A}'\Sigma_{\varepsilon}^{-1}\mathbf{y} \\
= & \mathbf{z}'\mathbf{B}\mathbf{z} - \mathbf{y}'\Sigma_{\varepsilon}^{-1}\mathbf{A}\mathbf{B}^{-1}\mathbf{B}\mathbf{z} - \mathbf{z}'\mathbf{A}'\Sigma_{\varepsilon}^{-1}\mathbf{y} + \mathbf{y}'\Sigma_{\varepsilon}^{-1}\mathbf{A}\mathbf{B}^{-1}\mathbf{B}\mathbf{B}^{-1}\mathbf{A}'\Sigma_{\varepsilon}^{-1}\mathbf{y} \\
= & (\mathbf{z} - \mathbf{B}^{-1}\mathbf{A}'\Sigma_{\varepsilon}^{-1}\mathbf{y})'\mathbf{B}(\mathbf{z} - \mathbf{B}^{-1}\mathbf{A}'\Sigma_{\varepsilon}^{-1}\mathbf{y})
\end{aligned}$$

Hence, the pdf of $\boldsymbol{\theta}$ given \mathbf{x} is

$$\begin{aligned}
f(\boldsymbol{\theta}|\mathbf{x}) &= \frac{(2\pi)^{-\frac{k}{2}}|\Sigma_{\varepsilon}|^{-\frac{1}{2}}|\Sigma_{\theta}|^{-\frac{1}{2}}}{|\mathbf{A}\Sigma_{\theta}\mathbf{A}' + \Sigma_{\varepsilon}|^{-\frac{1}{2}}} \exp\left[-\frac{1}{2}\left[(\mathbf{z} - \mathbf{B}^{-1}\mathbf{A}'\Sigma_{\varepsilon}^{-1}\mathbf{y})'\mathbf{B}(\mathbf{z} - \mathbf{B}^{-1}\mathbf{A}'\Sigma_{\varepsilon}^{-1}\mathbf{y})\right]\right] \\
&= \frac{(2\pi)^{-\frac{k}{2}}|\Sigma_{\varepsilon}|^{-\frac{1}{2}}|\Sigma_{\theta}|^{-\frac{1}{2}}}{|\mathbf{A}\Sigma_{\theta}\mathbf{A}' + \Sigma_{\varepsilon}|^{-\frac{1}{2}}} \\
&\quad \exp\left[-\frac{1}{2}\left[\left((\boldsymbol{\theta} - \boldsymbol{\mu}) - \mathbf{B}^{-1}\mathbf{A}'\Sigma_{\varepsilon}^{-1}(\mathbf{x} - \mathbf{A}\boldsymbol{\mu})\right)'\mathbf{B}\left((\boldsymbol{\theta} - \boldsymbol{\mu}) - \mathbf{B}^{-1}\mathbf{A}'\Sigma_{\varepsilon}^{-1}(\mathbf{x} - \mathbf{A}\boldsymbol{\mu})\right)\right]\right]
\end{aligned}$$

We can say that $\boldsymbol{\theta}|\mathbf{x} \sim \mathcal{N}(\boldsymbol{\mu} + \mathbf{B}^{-1}\mathbf{A}'\Sigma_{\varepsilon}^{-1}(\mathbf{x} - \mathbf{A}\boldsymbol{\mu}), \mathbf{B}^{-1})$. Let $\tilde{\boldsymbol{\mu}} = \boldsymbol{\mu} + \mathbf{B}^{-1}\mathbf{A}'\Sigma_{\varepsilon}^{-1}(\mathbf{x} - \mathbf{A}\boldsymbol{\mu})$. If $\boldsymbol{\theta}|\mathbf{x}$ is distributed as normal with mean $\boldsymbol{\mu} + \mathbf{B}^{-1}\mathbf{A}'\Sigma_{\varepsilon}^{-1}(\mathbf{x} - \mathbf{A}\boldsymbol{\mu})$ and variance \mathbf{B}^{-1} , then

$$\begin{aligned}
\int_{\mathbb{R}^k} (2\pi)^{-\frac{k}{2}}|\mathbf{B}^{-1}|^{-\frac{1}{2}} \exp\left(-\frac{1}{2}(\boldsymbol{\theta} - \tilde{\boldsymbol{\mu}})'\mathbf{B}(\boldsymbol{\theta} - \tilde{\boldsymbol{\mu}})\right) d\boldsymbol{\theta} &= 1 \\
\int_{\mathbb{R}^k} \exp\left(-\frac{1}{2}(\boldsymbol{\theta} - \tilde{\boldsymbol{\mu}})'\mathbf{B}(\boldsymbol{\theta} - \tilde{\boldsymbol{\mu}})\right) d\boldsymbol{\theta} &= (2\pi)^{\frac{k}{2}}|\mathbf{B}^{-1}|^{\frac{1}{2}}
\end{aligned}$$

If pdf $f(\boldsymbol{\theta}|\mathbf{x})$ is a normal pdf, then

$$\begin{aligned} \int_{\mathbb{R}^k} \frac{(2\pi)^{-\frac{k}{2}} |\Sigma_\varepsilon|^{-\frac{1}{2}} |\Sigma_\theta|^{-\frac{1}{2}}}{|\mathbf{A}\Sigma_\theta\mathbf{A}' + \Sigma_\varepsilon|^{-\frac{1}{2}}} \exp\left[-\frac{1}{2}(\boldsymbol{\theta} - \tilde{\boldsymbol{\mu}})' \mathbf{B}(\boldsymbol{\theta} - \tilde{\boldsymbol{\mu}})\right] &= 1 \\ \frac{(2\pi)^{-\frac{k}{2}} |\Sigma_\varepsilon|^{-\frac{1}{2}} |\Sigma_\theta|^{-\frac{1}{2}}}{|\mathbf{A}\Sigma_\theta\mathbf{A}' + \Sigma_\varepsilon|^{-\frac{1}{2}}} \int_{\mathbb{R}^k} \exp\left[-\frac{1}{2}(\boldsymbol{\theta} - \tilde{\boldsymbol{\mu}})' \mathbf{B}(\boldsymbol{\theta} - \tilde{\boldsymbol{\mu}})\right] d\boldsymbol{\theta} &= 1 \\ \frac{(2\pi)^{-\frac{k}{2}} |\Sigma_\varepsilon|^{-\frac{1}{2}} |\Sigma_\theta|^{-\frac{1}{2}}}{|\mathbf{A}\Sigma_\theta\mathbf{A}' + \Sigma_\varepsilon|^{-\frac{1}{2}}} (2\pi)^{\frac{k}{2}} |\mathbf{B}^{-1}|^{\frac{1}{2}} &= 1 \end{aligned}$$

Therefore, $\frac{|\Sigma_\varepsilon|^{-\frac{1}{2}} |\Sigma_\theta|^{-\frac{1}{2}}}{|\mathbf{A}\Sigma_\theta\mathbf{A}' + \Sigma_\varepsilon|^{-\frac{1}{2}}} = |\mathbf{B}^{-1}|^{-\frac{1}{2}}$

For signal extraction $\mathbf{x} = \mathbf{A}\boldsymbol{\theta} + \boldsymbol{\varepsilon}$, where $\boldsymbol{\theta} \sim \mathbf{N}(\boldsymbol{\mu}, \Sigma_\theta)$ and $\boldsymbol{\varepsilon} \sim \mathbf{N}(\mathbf{0}, \Sigma_\varepsilon)$ independent of $\boldsymbol{\theta}$, then $\boldsymbol{\theta}|\mathbf{x} \sim \mathbf{N}(\boldsymbol{\mu} + (\Sigma_\theta^{-1} + \mathbf{A}'\Sigma_\varepsilon^{-1}\mathbf{A})^{-1}\mathbf{A}'\Sigma_\varepsilon^{-1}(\mathbf{x} - \mathbf{A}\boldsymbol{\mu}), (\Sigma_\theta^{-1} + \mathbf{A}'\Sigma_\varepsilon^{-1}\mathbf{A})^{-1})$.

A.4 Expectation of ε_{t+1}^f and ε_{t+2}^f of Group i and j

Investors in group i can extract information about ε_{t+1}^f and ε_{t+2}^f from four signals; namely, $v_t^i - \rho_f^2 f_t$, $v_{t-1}^i - \rho_f f_t$, $\tilde{s}_t - \tilde{\gamma}_{-2} b_{t-2}$, and $\tilde{s}_{t-1} - \tilde{\lambda}_1 \varepsilon_t^f - \tilde{\gamma}_{-1} \varepsilon_{t-2}^b - \tilde{\gamma}_{-2} b_{t-3}$.

Define

$$\mathbf{x} = \begin{pmatrix} v_t^i - \rho_f^2 f_t \\ v_{t-1}^i - \rho_f f_t \\ \tilde{s}_t - \tilde{\gamma}_{-2} b_{t-2} \\ \tilde{s}_{t-1} - \tilde{\lambda}_1 \varepsilon_t^f - \tilde{\gamma}_{-1} \varepsilon_{t-2}^b - \tilde{\gamma}_{-2} b_{t-3} \end{pmatrix}, \mathbf{A} = \begin{pmatrix} \rho_f & 1 \\ 1 & 0 \\ \tilde{\lambda}_1 & \tilde{\lambda}_2 \\ \tilde{\lambda}_2 & 0 \end{pmatrix}$$

and

$$\boldsymbol{\theta} = \begin{pmatrix} \varepsilon_{t+1}^f \\ \varepsilon_{t+2}^f \end{pmatrix}, \boldsymbol{\varepsilon} = \begin{pmatrix} \varepsilon_t^{v_i} \\ \varepsilon_{t-1}^{v_i} \\ \tilde{\gamma}_{-1} \varepsilon_{t-1}^b + \tilde{\gamma}_0 \varepsilon_t^b \\ \tilde{\gamma}_0 \varepsilon_{t-1}^b \end{pmatrix}.$$

Then

$$\Sigma_\theta = \begin{pmatrix} \sigma_f^2 & 0 \\ 0 & \sigma_f^2 \end{pmatrix}, \text{ and } \Sigma_\varepsilon = \begin{pmatrix} \sigma_{v_i}^2 & 0 & 0 & 0 \\ 0 & \sigma_{v_i}^2 & 0 & 0 \\ 0 & 0 & \tilde{\gamma}_{-1}^2 \sigma_b^2 + \tilde{\gamma}_0^2 \sigma_b^2 & \tilde{\gamma}_{-1} \tilde{\gamma}_0 \sigma_b^2 \\ 0 & 0 & \tilde{\gamma}_{-1} \tilde{\gamma}_0 \sigma_b^2 & \tilde{\gamma}_0^2 \sigma_b^2 \end{pmatrix}.$$

Also let $\beta_{v_i} = \frac{1}{\sigma_{v_i}^2}$, $\beta_f = \frac{1}{\sigma_f^2}$, $\psi = \frac{1}{\tilde{\gamma}_0^2 \sigma_b^2}$, $\tilde{\tau} = \frac{\tilde{\gamma}_{-1}}{\tilde{\gamma}_0}$, $A = \beta_f + \beta_{v_i} + \tilde{\lambda}_2^2 \psi$, and $B = [\beta_f + \rho_f^2 \beta_{v_i} + \beta_{v_i} + \tilde{\lambda}_1 \psi (\tilde{\lambda}_1 - \tilde{\lambda}_2 \tilde{\tau}) + \tilde{\lambda}_2 \psi (\tilde{\lambda}_2 (1 + \tilde{\tau}^2) - \tilde{\lambda}_1 \tilde{\tau})][\beta_f + \beta_{v_i} + \tilde{\lambda}_2^2 \psi] - [\rho_f \beta_{v_i} + \tilde{\lambda}_2 \psi (\tilde{\lambda}_1 - \tilde{\lambda}_2 \tilde{\tau})]^2$. This leads to

$$\begin{aligned} E_t^i \varepsilon_{t+1}^f &= \frac{1}{B} \left\{ [A \rho_f - \rho_f \beta_{v_i} - \tilde{\lambda}_2 \psi (\tilde{\lambda}_1 - \tilde{\lambda}_2 \tilde{\tau})] \beta_{v_i} (v_t^i - \rho_f^2 f_t) + A \beta_{v_i} (v_{t-1}^i - \rho_f f_t) \right. \\ &\quad + [(\tilde{\lambda}_1 - \tilde{\lambda}_2 \tilde{\tau})(A - \tilde{\lambda}_2^2 \psi) \psi - \tilde{\lambda}_2 \psi \rho_f \beta_{v_i}] [\tilde{s}_t - \tilde{\gamma}_{-2} b_{t-2}] \\ &\quad \left. + [(\tilde{\lambda}_1 - \tilde{\lambda}_2 \tilde{\tau})(\tilde{\lambda}_2^2 \psi - A) \psi \tilde{\tau} + A \psi \tilde{\lambda}_2 + \rho_f \beta_{v_i} \tilde{\lambda}_2 \tilde{\tau} \psi] [\tilde{s}_{t-1} - \tilde{\lambda}_1 \varepsilon_t^f - \tilde{\gamma}_{-1} \varepsilon_{t-2}^b - \tilde{\gamma}_{-2} b_{t-3}] \right\}, \end{aligned}$$

and

$$\begin{aligned} E_t^i \varepsilon_{t+2}^f &= \frac{1}{B} \left\{ [[-\rho_f \beta_{v_i} - \tilde{\lambda}_2 \psi (\tilde{\lambda}_1 - \tilde{\lambda}_2 \tilde{\tau})] \rho_f + A + \rho_f^2 \beta_{v_i} + (\tilde{\lambda}_1 - \tilde{\lambda}_2 \tilde{\tau})^2 \psi] \beta_{v_i} (v_t^i - \rho_f^2 f_t) \right. \\ &\quad + [-\rho_f \beta_{v_i} - \tilde{\lambda}_2 \psi (\tilde{\lambda}_1 - \tilde{\lambda}_2 \tilde{\tau})] \beta_{v_i} (v_{t-1}^i - \rho_f f_t) \\ &\quad + [-\psi (\tilde{\lambda}_1 - \tilde{\lambda}_2 \tilde{\tau}) \rho_f \beta_{v_i} + (A + \rho_f^2 \beta_{v_i}) \tilde{\lambda}_2 \psi] [\tilde{s}_t - \tilde{\gamma}_{-2} b_{t-2}] \\ &\quad + [\rho_f \beta_{v_i} \psi (\tilde{\tau} (\tilde{\lambda}_1 - \tilde{\lambda}_2 \tilde{\tau}) - \tilde{\lambda}_2) - A \tilde{\lambda}_2 \tilde{\tau} \psi - \tilde{\lambda}_2^2 \psi^2 (\tilde{\lambda}_1 - \tilde{\lambda}_2 \tilde{\tau}) - \rho_f^2 \beta_{v_i} \tilde{\lambda}_2 \tilde{\tau} \psi] \times \\ &\quad \left. [\tilde{s}_{t-1} - \tilde{\lambda}_1 \varepsilon_t^f - \tilde{\gamma}_{-1} \varepsilon_{t-2}^b - \tilde{\gamma}_{-2} b_{t-3}] \right\} \end{aligned}$$

Group j investors receive three signals from $v_t^j - \rho_f f_t$, $\tilde{s}_t - \tilde{\gamma}_{-2} b_{t-2}$, and $\tilde{s}_{t-1} - \tilde{\lambda}_1 \varepsilon_t^f - \tilde{\gamma}_{-1} \varepsilon_{t-2}^b - \tilde{\gamma}_{-2} b_{t-3}$. With these signals, they extract information about ε_{t+1}^f and ε_{t+2}^f . Although investors j do not receive any private signal about ε_{t+2}^f , they will know about it from the exchange rate signal.

Now let

$$\mathbf{x} = \begin{pmatrix} v_t^j - \rho_f f_t \\ \tilde{s}_t - \tilde{\gamma}_{-2} b_{t-2} \\ \tilde{s}_{t-1} - \tilde{\lambda}_1 \varepsilon_t^f - \tilde{\gamma}_{-1} \varepsilon_{t-2}^b - \tilde{\gamma}_{-2} b_{t-3} \end{pmatrix}, \mathbf{A} = \begin{pmatrix} 1 & 0 \\ \tilde{\lambda}_1 & \tilde{\lambda}_2 \\ \tilde{\lambda}_2 & 0 \end{pmatrix},$$

and

$$\boldsymbol{\theta} = \begin{pmatrix} \varepsilon_{t+1}^f \\ \varepsilon_{t+2}^f \end{pmatrix}, \boldsymbol{\varepsilon} = \begin{pmatrix} \varepsilon_t^j \\ \tilde{\gamma}_{-1} \varepsilon_{t-1}^b + \tilde{\gamma}_0 \varepsilon_t^b \\ \tilde{\gamma}_0 \varepsilon_{t-1}^b \end{pmatrix}.$$

Then

$$\boldsymbol{\Sigma}_\theta = \begin{pmatrix} \sigma_f^2 & 0 \\ 0 & \sigma_f^2 \end{pmatrix}, \boldsymbol{\Sigma}_\varepsilon = \begin{pmatrix} \sigma_{v_j}^2 & 0 & 0 \\ 0 & \tilde{\gamma}_{-1}^2 \sigma_b^2 + \tilde{\gamma}_0^2 \sigma_b^2 & \tilde{\gamma}_{-1} \tilde{\gamma}_0 \sigma_b^2 \\ 0 & \tilde{\gamma}_{-1} \tilde{\gamma}_0 \sigma_b^2 & \tilde{\gamma}_0^2 \sigma_b^2 \end{pmatrix}.$$

Define $\beta_{v_j} = \frac{1}{\sigma_{v_j}^2}$, $C = \beta_f + \tilde{\lambda}_2^2 \psi$, and $D = [\beta_f + \beta_{v_j} + \tilde{\lambda}_2^2 \psi + (\tilde{\lambda}_1 - \tilde{\lambda}_2 \tilde{\tau})^2 \psi][\beta_f + \tilde{\lambda}_2^2 \psi] - \tilde{\lambda}_2^2 \psi^2 (\tilde{\lambda}_1 - \tilde{\lambda}_2 \tilde{\tau})^2$. One can compute

$$E_t^j \varepsilon_{t+1}^f = \frac{1}{D} \left\{ C \beta_{v_j} (v_t^j - \rho_f f_t) + (\tilde{\lambda}_1 - \tilde{\lambda}_2 \tau) \psi \beta_f (\tilde{s}_t - \tilde{\gamma}_{-2} b_{t-2}) \right. \\ \left. + \psi (C \tilde{\lambda}_2 - \tilde{\tau} (\tilde{\lambda}_1 - \tilde{\lambda}_2 \tau) \beta_f) (\tilde{s}_{t-1} - \tilde{\lambda}_1 \varepsilon_t^f - \tilde{\gamma}_{-1} \varepsilon_{t-2}^b - \tilde{\gamma}_{-2} b_{t-3}) \right\},$$

and

$$E_t^j \varepsilon_{t+2}^f = \frac{1}{D} \left\{ -\tilde{\lambda}_2 \psi (\tilde{\lambda}_1 - \tilde{\lambda}_2 \tilde{\tau}) \beta_{v_j} (v_t^j - \rho_f f_t) + \tilde{\lambda}_2 \psi (C + \beta_{v_j}) (\tilde{s}_t - \tilde{\gamma}_{-2} b_{t-2}) \right. \\ \left. + [-\tilde{\lambda}_2^2 \psi^2 (\tilde{\lambda}_1 - \tilde{\lambda}_2 \tilde{\tau}) - (C + \beta_{v_j}) \tilde{\lambda}_2 \psi \tilde{\tau}] [\tilde{s}_{t-1} - \tilde{\lambda}_1 \varepsilon_t^f - \tilde{\gamma}_{-1} \varepsilon_{t-2}^b - \tilde{\gamma}_{-2} b_{t-3}] \right\}$$

Define $a_1 = \frac{1}{B} [A \rho_f - \rho_f \beta_{v_i} - \tilde{\lambda}_2 \psi (\tilde{\lambda}_1 - \tilde{\lambda}_2 \tau)] \beta_{v_i}$, $a_2 = \frac{1}{B} A \beta_{v_i}$, $a_3 = \frac{1}{B} [(\tilde{\lambda}_1 - \tilde{\lambda}_2 \tilde{\tau}) (A - \tilde{\lambda}_2^2 \psi) \psi - \tilde{\lambda}_2 \psi \rho_f \beta_{v_i}]$, $a_4 = -a_3 \tilde{\gamma}_{-2}$, $a_5 = -[a_1 \rho_f^2 + a_2 \rho_f + a_3 \tilde{\lambda}_0]$, $a_6 = \frac{1}{B} [(\tilde{\lambda}_1 - \tilde{\lambda}_2 \tilde{\tau}) (\tilde{\lambda}_2^2 \psi - A) \psi \tilde{\tau} + A \psi \tilde{\lambda}_2 + \rho_f \beta_{v_i} \tilde{\lambda}_2 \tilde{\tau} \psi] \hat{s}_{t-1}$, where $\hat{s}_{t-1} = \tilde{s}_{t-1} - \tilde{\lambda}_1 \varepsilon_t^f - \tilde{\gamma}_{-1} \varepsilon_{t-2}^b - \tilde{\gamma}_{-2} b_{t-3}$, $c_1 = \frac{1}{B} [-\rho_f \beta_{v_i} - \tilde{\lambda}_2 \psi (\tilde{\lambda}_1 - \tilde{\lambda}_2 \tau)] \rho_f + A + \rho_f^2 \beta_{v_i} + (\tilde{\lambda}_1 - \tilde{\lambda}_2 \tilde{\tau})^2 \psi \beta_{v_i}$, $c_2 = \frac{1}{B} [-\rho_f \beta_{v_i} - \tilde{\lambda}_2 \psi (\tilde{\lambda}_1 - \tilde{\lambda}_2 \tilde{\tau})] \beta_{v_i}$, $c_3 = \frac{1}{B} [(A + \rho_f^2 \beta_{v_i}) \tilde{\lambda}_2 \psi - \psi (\tilde{\lambda}_1 - \tilde{\lambda}_2 \tilde{\tau}) \rho_f \beta_{v_i}]$, $c_4 = -c_3 \tilde{\gamma}_{-2}$, $c_5 = -[c_1 \rho_f^2 + c_2 \rho_f + c_3 \tilde{\lambda}_0]$, $c_6 = \frac{1}{B} [\rho_f \beta_{v_i} \psi [\tilde{\tau} (\tilde{\lambda}_1 - \tilde{\lambda}_2 \tilde{\tau}) - \tilde{\lambda}_2] - A \tilde{\lambda}_2 \tilde{\tau} \psi - \tilde{\lambda}_2^2 \psi^2 (\tilde{\lambda}_1 - \tilde{\lambda}_2 \tilde{\tau}) - \rho_f^2 \beta_{v_i} \tilde{\lambda}_2 \tilde{\tau} \psi]$, $d_1 = \frac{C}{D} \beta_{v_j}$, $d_3 = \frac{\psi \beta_f (\tilde{\lambda}_1 - \tilde{\lambda}_2 \tilde{\tau})}{D}$, $d_4 = -d_3 \tilde{\gamma}_{-2}$, $d_5 = -[d_1 \rho_f + d_3 \tilde{\lambda}_0]$, $d_6 = \frac{\psi}{D} [C \tilde{\lambda}_2 - \tilde{\tau} \beta_f (\tilde{\lambda}_1 - \tilde{\lambda}_2 \tilde{\tau})]$, $e_1 = -\frac{\tilde{\lambda}_2 \psi}{D} (\tilde{\lambda}_1 - \tilde{\lambda}_2 \tilde{\tau}) \beta_{v_j}$, $e_3 = \frac{(C + \beta_{v_j}) \tilde{\lambda}_2 \psi}{D}$, $e_4 = -e_3 \tilde{\gamma}_{-2}$, $e_5 = -[e_1 \rho_f + e_3 \tilde{\lambda}_0]$, $e_6 = -\frac{1}{D} [\tilde{\lambda}_2^2 \psi^2 (\tilde{\lambda}_1 - \tilde{\lambda}_2 \tilde{\tau}) + (C + \beta_{v_j}) \tilde{\lambda}_2 \psi \tilde{\tau}]$. Therefore, I can write

$$E_t^i \varepsilon_{t+1}^f = a_1 v_t^i + a_2 v_{t-1}^i + a_3 s_t + a_4 b_{t-2} + a_5 f_t + a_6 \hat{s}_{t-1}$$

$$E_t^i \varepsilon_{t+2}^f = c_1 v_t^i + c_2 v_{t-1}^i + c_3 s_t + c_4 b_{t-2} + c_5 f_t + c_6 \hat{s}_{t-1}$$

$$E_t^j \varepsilon_{t+1}^f = d_1 v_t^j + d_3 s_t + d_4 b_{t-2} + d_5 f_t + d_6 \hat{s}_{t-1}$$

$$E_t^j \varepsilon_{t+2}^f = e_1 v_t^j + e_3 s_t + e_4 b_{t-2} + e_5 f_t + e_6 \hat{s}_{t-1}$$

A.5 Expectation of ε_{t-1}^b and ε_t^b of Group i and j

Recall that exchange rate signal $\hat{s}_{t-1} = \tilde{s}_{t-1} - \tilde{\lambda}_1 \varepsilon_t^f - \tilde{\gamma}_{-1} \varepsilon_{t-2}^b - \tilde{\gamma}_{-2} b_{t-3} = \tilde{\lambda}_2 \varepsilon_{t+1}^f + \tilde{\gamma}_0 \varepsilon_{t-1}^b$, then

I can write

$$\begin{aligned} E_t^i \varepsilon_{t-1}^b &= \frac{\hat{s}_{t-1}}{\tilde{\gamma}_0} - \frac{\tilde{\lambda}_2}{\tilde{\gamma}_0} E_t^i \varepsilon_{t+1}^f \\ &= \frac{\hat{s}_{t-1}}{\tilde{\gamma}_0} - \frac{\tilde{\lambda}_2}{\tilde{\gamma}_0} [a_1 v_t^i + a_2 v_{t-1}^i + a_3 s_t + a_4 b_{t-2} + a_5 f_t + a_6 \hat{s}_{t-1}] \\ &= g_1 v_t^i + g_2 v_{t-1}^i + g_3 s_t + g_4 b_{t-2} + g_5 f_t + g_6 \hat{s}_{t-1}, \end{aligned}$$

where $g_1 = -\frac{\tilde{\lambda}_2}{\tilde{\gamma}_0} a_1$, $g_2 = -\frac{\tilde{\lambda}_2}{\tilde{\gamma}_0} a_2$, $g_3 = -\frac{\tilde{\lambda}_2}{\tilde{\gamma}_0} a_3$, $g_4 = -\frac{\tilde{\lambda}_2}{\tilde{\gamma}_0} a_4$, $g_5 = -\frac{\tilde{\lambda}_2}{\tilde{\gamma}_0} a_5$, and $g_6 = \left[\frac{1}{\tilde{\gamma}_0} - \frac{\tilde{\lambda}_2}{\tilde{\gamma}_0} a_6 \right]$.

Repeat the same method for $E_t^j \varepsilon_{t-1}^b$, this yields

$$E_t^j \varepsilon_{t-1}^b = h_1 v_t^j + h_3 s_t + h_4 b_{t-2} + h_5 f_t + h_6 \hat{s}_{t-1}$$

where $h_1 = -\frac{\tilde{\lambda}_2}{\tilde{\gamma}_0} d_1$, $h_3 = -\frac{\tilde{\lambda}_2}{\tilde{\gamma}_0} d_3$, $h_4 = -\frac{\tilde{\lambda}_2}{\tilde{\gamma}_0} d_4$, $h_5 = -\frac{\tilde{\lambda}_2}{\tilde{\gamma}_0} d_5$, and $h_6 = \left[\frac{1}{\tilde{\gamma}_0} - \frac{\tilde{\lambda}_2}{\tilde{\gamma}_0} d_6 \right]$.

I know that $\tilde{s}_t - \tilde{\gamma}_{-2} b_{t-2} = \tilde{\lambda}_1 E_t^i \varepsilon_{t+1}^f + \tilde{\lambda}_2 E_t^i \varepsilon_{t+2}^f + \tilde{\gamma}_0 E_t^i \varepsilon_t^b + \tilde{\gamma}_{-1} E_t^i \varepsilon_{t-1}^b$. Then I can write

$$\begin{aligned} E_t^i \varepsilon_t^b &= \frac{s_t}{\tilde{\gamma}_0} - \frac{\tilde{\lambda}_0}{\tilde{\gamma}_0} f_t - \frac{\tilde{\gamma}_{-2}}{\tilde{\gamma}_0} b_{t-2} - \frac{\tilde{\lambda}_1}{\tilde{\gamma}_0} E_t^i \varepsilon_{t+1}^f - \frac{\tilde{\lambda}_2}{\tilde{\gamma}_0} E_t^i \varepsilon_{t+2}^f - \frac{\tilde{\gamma}_{-1}}{\tilde{\gamma}_0} E_t^i \varepsilon_{t-1}^b \\ &= k_1 v_t^i + k_2 v_{t-1}^i + k_3 s_t + k_4 b_{t-2} + k_5 f_t + k_6 \hat{s}_{t-1}, \end{aligned}$$

where

$$\begin{aligned}
 k_1 &= \left[-\frac{\tilde{\lambda}_1}{\tilde{\gamma}_0} a_1 - \frac{\tilde{\lambda}_2}{\tilde{\gamma}_0} c_1 - \frac{\tilde{\gamma}_{-1}}{\tilde{\gamma}_0} g_1 \right], \\
 k_2 &= \left[-\frac{\tilde{\lambda}_1}{\tilde{\gamma}_0} a_2 - \frac{\tilde{\lambda}_2}{\tilde{\gamma}_0} c_2 - \frac{\tilde{\gamma}_{-1}}{\tilde{\gamma}_0} g_2 \right], \\
 k_3 &= \left[\frac{1}{\tilde{\gamma}_0} - \frac{\tilde{\lambda}_1}{\tilde{\gamma}_0} a_3 - \frac{\tilde{\lambda}_2}{\tilde{\gamma}_0} c_3 - \frac{\tilde{\gamma}_{-1}}{\tilde{\gamma}_0} g_3 \right], \\
 k_4 &= \left[-\frac{\tilde{\gamma}_{-2}}{\tilde{\gamma}_0} - \frac{\tilde{\lambda}_1}{\tilde{\gamma}_0} a_4 - \frac{\tilde{\lambda}_2}{\tilde{\gamma}_0} c_4 - \frac{\tilde{\gamma}_{-1}}{\tilde{\gamma}_0} g_4 \right], \\
 k_5 &= \left[-\frac{\tilde{\lambda}_0}{\tilde{\gamma}_0} - \frac{\tilde{\lambda}_1}{\tilde{\gamma}_0} a_5 - \frac{\tilde{\lambda}_2}{\tilde{\gamma}_0} c_5 - \frac{\tilde{\gamma}_{-1}}{\tilde{\gamma}_0} g_5 \right], \\
 k_6 &= \left[-\frac{\tilde{\lambda}_1}{\tilde{\gamma}_0} a_6 - \frac{\tilde{\lambda}_2}{\tilde{\gamma}_0} c_6 - \frac{\tilde{\gamma}_{-1}}{\tilde{\gamma}_0} g_6 \right].
 \end{aligned}$$

Also, I can write

$$E_t^j \varepsilon_t^b = m_1 v_t^j + m_3 s_t + m_4 b_{t-2} + m_5 f_t + m_6 \hat{s}_{t-1},$$

where

$$\begin{aligned}
 m_1 &= \left[-\frac{\tilde{\lambda}_1}{\tilde{\gamma}_0} d_1 - \frac{\tilde{\lambda}_2}{\tilde{\gamma}_0} e_1 - \frac{\tilde{\gamma}_{-1}}{\tilde{\gamma}_0} h_1 \right], \\
 m_3 &= \left[\frac{1}{\tilde{\gamma}_0} - \frac{\tilde{\lambda}_1}{\tilde{\gamma}_0} d_3 - \frac{\tilde{\lambda}_2}{\tilde{\gamma}_0} e_3 - \frac{\tilde{\gamma}_{-1}}{\tilde{\gamma}_0} h_3 \right], \\
 m_4 &= \left[-\frac{\tilde{\gamma}_{-2}}{\tilde{\gamma}_0} - \frac{\tilde{\lambda}_1}{\tilde{\gamma}_0} d_4 - \frac{\tilde{\lambda}_2}{\tilde{\gamma}_0} e_4 - \frac{\tilde{\gamma}_{-1}}{\tilde{\gamma}_0} h_4 \right], \\
 m_5 &= \left[-\frac{\tilde{\lambda}_0}{\tilde{\gamma}_0} - \frac{\tilde{\lambda}_1}{\tilde{\gamma}_0} d_5 - \frac{\tilde{\lambda}_2}{\tilde{\gamma}_0} e_5 - \frac{\tilde{\gamma}_{-1}}{\tilde{\gamma}_0} h_5 \right], \\
 m_6 &= \left[-\frac{\tilde{\lambda}_1}{\tilde{\gamma}_0} d_6 - \frac{\tilde{\lambda}_2}{\tilde{\gamma}_0} e_6 - \frac{\tilde{\gamma}_{-1}}{\tilde{\gamma}_0} h_6 \right].
 \end{aligned}$$

APPENDIX B. CHAPTER 2 APPENDIX

B.1 Log-Linearized System

Interest Rate Firstly, I will show the log-linearized equation of the demand for home bond. Let us begin with the home bond demand equation (2.3). Multiplying and dividing by \bar{R} , \bar{C} , and \bar{P} leads to

$$1 = \frac{R_t}{\bar{R}} E_t \beta \bar{R} \left(\frac{C_{t+1}/\bar{C}}{C_t/\bar{C}} \right)^{-\sigma} \left(\frac{P_t/\bar{P}}{P_{t+1}/\bar{P}} \right).$$

Taking log both sides of the equation yields

$$0 = (\log R_t - \log \bar{R}) + E_t [-\sigma [(\log C_{t+1} - \log \bar{C}) - (\log C_t - \log \bar{C})] + (\log P_t - \log \bar{P}) - (\log P_{t+1} - \log \bar{P})].$$

Finally, I can write the log-linearized of home interest rate as

$$i_t = E_t [\sigma \Delta c_{t+1} + \Delta p_{t+1}].$$

Repeat the same process on the households' demand for foreign bond, equation (2.7), I will have the log-linearized foreign bond as in equation (2.14).

Country Budget Constraint The linear deviation for the home budget constraint can be computed by firstly multiplying \bar{R}/\bar{Y} on both sides of equation (2.6).

$$\begin{aligned} \frac{\bar{R}B_{t+1}}{\bar{Y}} - \frac{\bar{R}R_{t-1}B_t}{\bar{Y}} &= \frac{\bar{R}NX_t}{\bar{Y}} \\ \frac{b_{t+1}}{\bar{R}} - \frac{R_{t-1}b_t}{\bar{R}} &= nx_t \end{aligned}$$

Since $\frac{R_{t-1}}{\bar{R}} = e^{\log R_{t-1} - \log \bar{R}} \approx 1 + i_{t-1}$, then the above equation becomes

$$\beta b_{t+1} - (1 + i_{t-1})b_t = nx_t.$$

I drop the term $i_{t-1}b_t$ because it is small when comparing to the first term, b_t . Hence, the loglinearization of the home budget constraint is

$$\beta b_{t+1} - b_t = nx_t.$$

For the foreign country budget constraint, I begin by multiplying \bar{R}/\bar{Y} on both sides of equation (2.10).

$$\begin{aligned}\frac{\bar{R}B_{t+1}^*}{\bar{Y}} - \frac{\bar{R}R_{t-1}B_t^*}{\bar{Y}} &= \frac{\bar{R}NX_t^*}{\bar{Y}} + \frac{\bar{R}\tilde{R}_t(N_t^* + D_t^*)}{\bar{Y}} \\ \frac{b_{t+1}}{\bar{R}} - \frac{R_{t-1}b_t}{\bar{R}} &= nx_t + \frac{\tilde{R}_t(N_t^* + D_t^*)}{\bar{Y}}\end{aligned}$$

Define $\tilde{n}_t^* = N_t^*/\bar{Y}$ and $\tilde{d}_t^* = D_t^*/\bar{Y}$. I can show that $\tilde{R}_t(N_t^* + D_t^*)/\bar{Y}$ approximates as zero. Since

$$\frac{\tilde{R}_t(N_t^* + D_t^*)}{\bar{Y}} = \left[R_{t-1}^* - R_{t-1} \frac{S_{t-1}}{S_t} \right] (\tilde{n}_t^* + \tilde{d}_t^*),$$

the right hand side term is distributed to four terms, which are:

$$\begin{aligned}R_{t-1}^* \tilde{n}_t^* &\approx \frac{(1 + i_{t-1}^*) \tilde{n}_t^*}{\beta} \\ R_{t-1} \frac{S_{t-1}}{S_t} \tilde{n}_t^* &\approx \frac{(1 + i_{t-1} + s_{t-1} - s_t) \tilde{n}_t^*}{\beta} \\ R_{t-1}^* \tilde{d}_t^* &\approx \frac{(1 + i_{t-1}^*) \tilde{d}_t^*}{\beta} \\ R_{t-1} \frac{S_{t-1}}{S_t} \tilde{d}_t^* &\approx \frac{(1 + i_{t-1} + s_{t-1} - s_t) \tilde{d}_t^*}{\beta}.\end{aligned}$$

Thus,

$$\frac{\tilde{R}_t(N_t^* + D_t^*)}{\bar{Y}} \approx \frac{(1 + i_{t-1}^*) \tilde{n}_t^*}{\beta} - \frac{(1 + i_{t-1} + s_{t-1} - s_t) \tilde{n}_t^*}{\beta} + \frac{(1 + i_{t-1}^*) \tilde{d}_t^*}{\beta} - \frac{(1 + i_{t-1} + s_{t-1} - s_t) \tilde{d}_t^*}{\beta}$$

I drop $i_{t-1}^* \tilde{n}_t^*$, $(i_{t-1} + s_{t-1} - s_t) \tilde{n}_t^*$, $i_{t-1}^* \tilde{d}_t^*$, and $(i_{t-1} + s_{t-1} - s_t) \tilde{d}_t^*$ because their values are small.

Therefore, $\frac{\tilde{R}_t(N_t^* + D_t^*)}{\bar{Y}} \approx 0$.

The foreign country budget constraint is thus written as

$$\beta b_{t+1}^* - b_t^* = nx_t^*.$$

Real Exchange Rates and Prices The log real exchange rate and terms of trade are given

by

$$\begin{aligned}q_t &= p_t^* + s_t - p_t, \\ z_t &= p_{Ft} - p_{Ht}^* - s_t.\end{aligned}$$

The law of one price can be written as $Q_{Ht} \equiv \frac{P_{Ht}^* S_t}{P_{Ht}} = Q_t^\alpha$, where $Q_t \equiv \frac{P_t^* S_t}{P_t}$. Define the log of law of one price deviations and its foreign counterpart as:

$$\begin{aligned} q_{Ht} &\equiv p_{Ht}^* + s_t - p_{Ht} = \alpha q_t, \\ q_{Ft} &\equiv p_{Ft}^* + s_t - p_{Ft} = \alpha q_t. \end{aligned}$$

I rewrite the expression for z_t as follows:

$$\begin{aligned} z_t &= p_{Ft} - p_{Ht}^* - s_t \\ &= (p_{Ft}^* + s_t - \alpha q_t) - (\alpha q_t - s_t + p_{Ht}) - s_t \\ &= p_{Ft}^* + s_t - p_{Ht} - 2\alpha q_t. \end{aligned}$$

The producer-price-based real exchange rate is q_t^P , defined as $q_t^P = p_{Ft}^* + s_t - p_{Ht}$. Thus I obtain

$$z_t = q_t^P - 2\alpha q_t.$$

The expression for q_t can be written as

$$\begin{aligned} q_t &= p_t^* + s_t - p_t \\ &= \gamma p_{Ht}^* + (1 - \gamma) p_{Ft}^* + s_t - (1 - \gamma) p_{Ht} - \gamma p_{Ft} \\ &= (1 - \gamma) q_t^P - \gamma z_t. \end{aligned}$$

Thus, I can solve for q_t^P and z_t as a function of q_t :

$$\begin{aligned} q_t^P &= \frac{q_t}{1 - \gamma} + \frac{\gamma z_t}{1 - \gamma} \\ &= \frac{q_t}{1 - \gamma} + \frac{\gamma}{1 - \gamma} (q_t^P - 2\alpha q_t) \\ &= \frac{1 - 2\alpha\gamma}{1 - 2\gamma} q_t \\ \\ z_t &= q_t^P - 2\alpha q_t \\ &= \frac{q_t + \gamma z_t}{1 - \gamma} - 2\alpha q_t \frac{(1 - \gamma)}{(1 - \gamma)} \\ &= \frac{1 - 2\alpha(1 - \gamma)}{1 - 2\gamma} q_t. \end{aligned}$$

Next, I use these results to solve for $p_{Ht} - p_t$ and $p_{Ft}^* - p_t^*$.

$$\begin{aligned}
 p_{Ht} - p_t &= p_{Ht} - (1 - \gamma)p_{Ht} - \gamma p_{Ft} \\
 &= \gamma(p_{Ht} - p_{Ft}) \\
 &= \gamma[-(p_{Ft} - p_{Ht}^* - s_t) - (p_{Ht}^* + s_t - p_{Ht})] \\
 &= \gamma[-(z_t + q_{Ht})] \\
 &= -\frac{(1 - \alpha)\gamma}{1 - 2\gamma}q_t \\
 p_{Ft}^* - p_t^* &= \gamma(p_{Ft}^* - p_{Ht}^*) = \frac{(1 - \alpha)\gamma}{1 - 2\gamma}q_t
 \end{aligned}$$

Then I can solve for the price levels:

$$\begin{aligned}
 p_t &= (1 - \gamma)p_{Ht} + \gamma p_{Ft} = p_{Ht} - \gamma(p_{Ht} - p_{Ft}) \\
 &= w_t + \frac{1}{1 - \phi} \frac{\gamma}{1 - 2\gamma} q_t, \\
 p_t^* &= w_t^* - \frac{1}{1 - \phi} \frac{\gamma}{1 - 2\gamma} q_t.
 \end{aligned}$$

Assume $w_t = w_t^* = 0$. Using the price levels to solve for q_t , I obtain:

$$q_t = \frac{1}{1 + \frac{1}{1 - \phi} \frac{2\gamma}{1 - 2\gamma}} s_t.$$

Real Exchange Rates and Quantities I log-linearize labor supply and labor demand equation as:

$$\begin{aligned}
 \sigma c_t + \frac{1}{\nu} l_t &= w_t - p_t \\
 l_t &= -\phi(w_t - p_t) + y_t.
 \end{aligned}$$

Combine these two equations, I have:

$$\nu \sigma c_t + y_t = -\frac{\nu + \phi}{1 - \phi} \frac{\gamma}{1 - 2\gamma} q_t.$$

The expression for foreign is:

$$\nu \sigma c_t^* + y_t^* = \frac{\nu + \phi}{1 - \phi} \frac{\gamma}{1 - 2\gamma} q_t.$$

For variable (x_t, x_t^*) , define $\tilde{x} \equiv (x_t - x_t^*)/2$. I can derive

$$\nu\sigma\tilde{c}_t + \tilde{y}_t = -\frac{\nu + \phi}{1 - \phi} \frac{\gamma}{1 - 2\gamma} q_t.$$

The log linearization of the quantities system is as follows:

$$\begin{aligned} y_t &= (1 - \gamma)y_{Ht} + \gamma y_{Ht}^*, \\ y_{Ht} &= -\theta(p_{Ht} - p_t) + \zeta c_t + (1 - \zeta)[(1 - \phi)(w_t - p_t + y_t)], \\ y_{Ht}^* &= -\theta(p_{Ht}^* - p_t^*) + \zeta c_t^* + (1 - \zeta)[(1 - \phi)(w_t^* - p_t^* + y_t^*)], \\ y_t^* &= (1 - \gamma)y_{Ft}^* + \gamma y_{Ft}, \\ y_{Ft}^* &= -\theta(p_{Ft}^* - p_t^*) + \zeta c_t^* + (1 - \zeta)[(1 - \phi)(w_t^* - p_t^* + y_t^*)], \\ y_{Ft} &= -\theta(p_{Ft} - p_t) + \zeta c_t + (1 - \zeta)[(1 - \phi)(w_t - p_t + y_t)], \end{aligned}$$

where $\zeta \equiv \bar{C}/\bar{Y}$. Replacing y_{Ht} and y_{Ht}^* into y_t , solving out for $(w_t - p_t)$, $(w_t^* - p_t^*)$, $p_{Ht} - p_t$, and $(p_{Ht}^* - p_t^*)$, I derive

$$y_t - (1 - \zeta)[y_t - 2\gamma\tilde{y}_t] - \zeta[c_t - 2\gamma\tilde{c}_t] = \gamma \left(\theta(1 - \alpha) \frac{2(1 - \gamma)}{1 - 2\gamma} - (1 - \zeta) \right) q_t.$$

Subtracting the foreign counterpart, I obtain:

$$[1 - (1 - 2\gamma)(1 - \zeta)]\tilde{y}_t - (1 - 2\gamma)\zeta\tilde{c}_t = \gamma \left(2\theta(1 - \alpha) \frac{1 - \gamma}{1 - 2\gamma} - (1 - \zeta) \right) q_t.$$

I use (B.1) and (B.1) to solve for consumption and output:

$$c_t - c_t^* = -\gamma\kappa_q^c q_t,$$

$$y_t - y_t^* = \gamma\kappa_q^y q_t,$$

where $\kappa_q^c = \frac{2\theta(1-\alpha)(1-\gamma) + \nu + \frac{\nu+\phi}{1-\phi} \frac{2\gamma}{1-2\gamma}}{1 + \sigma\nu[1 + \frac{1}{1-\phi} \frac{2\gamma}{1-2\gamma}]}$ and $\kappa_q^y = \sigma\nu\kappa_q^c - \frac{\nu+\phi}{1-\phi} \frac{2}{1-2\gamma}$.

The linearized expression for net exports is $nx_t = \gamma(y_{Ht}^* - y_{Ft} - s_t)$. Substituting for s_t , y_{Ht}^* , and y_{Ft} , I derive:

$$\begin{aligned} nx_t &= \gamma \left(\theta(1 - \alpha) \frac{2(1 - \gamma)}{1 - 2\gamma} + \alpha - \frac{1 - \alpha}{1 - 2\gamma} + \frac{2\gamma(1 - \zeta)}{1 - 2\gamma} \right) q_t - 2\gamma((1 - \phi)\tilde{c}_t + \phi\tilde{y}_t) \\ &= \gamma\kappa_q^{nx} q_t \\ &= \gamma\lambda_2 s_t, \end{aligned}$$

where $\lambda_2 = \frac{\kappa_q^{nx}}{1 + \frac{1}{1-\phi} \frac{2\gamma}{1-2\gamma}}$, $\kappa_q^{nx} = \left[2\theta(1-\alpha) \frac{1-\gamma}{1-2\gamma} + 2(1-\gamma)\alpha - 1 - \gamma\kappa_q^y \right] \frac{1}{1-2\gamma}$, and $\kappa_q^y = \sigma\nu\kappa_q^c - \frac{\nu+\phi}{1-\phi} \frac{2}{1-2\gamma}$.

B.2 UIP Condition

To derive the UIP condition, I start from market clearing condition:

$$\frac{B_{t+1}}{S_t} = n(e^{\psi_t} - 1) + \frac{mE_t\tilde{R}_{t+1}}{\omega var_t\tilde{R}_{t+1}}$$

Using the fact that

$$\tilde{R}_{t+1} = R_t^* \left(1 - \frac{R_t}{R_t^*} \frac{S_t}{S_{t+1}} \right) = R_t^* (1 - e^{i_t - i_t^* - \Delta s_{t+1}})$$

I can rewrite the equation as:

$$\frac{R_t^* B_{t+1}}{S_t} = R_t^* n(e^{\psi_t} - 1) + \frac{mE_t[1 - e^{i_t - i_t^* - \Delta s_{t+1}}]}{\omega var_t[1 - e^{i_t - i_t^* - \Delta s_{t+1}}]}$$

Using $\log x \approx x - 1$, the equation becomes

$$\frac{R_t^* B_{t+1}}{S_t} = R_t^* n\psi_t - \frac{m(i_t - i_t^* - E_t\Delta s_{t+1})}{\omega\sigma_s^2},$$

where $\sigma_s^2 = var_t(\Delta s_{t+1})$. Rearrange the equation in terms of b_{t+1} , I have

$$\frac{R_t^* \bar{P}\bar{Y}b_{t+1}}{\bar{R} S_t} = \frac{R_t^* n}{\bar{R} \beta} \psi_t - \frac{m}{\omega\sigma_s^2} (i_t - i_t^* - E_t\Delta s_{t+1}).$$

Since $\frac{R_t^*}{\bar{R} S_t} \approx 1 + i_t^* - s_t$, and $\frac{R_t^*}{\bar{R}} \approx 1 + i_t^*$, I can write

$$(1 + i_t^* - s_t) \bar{P}\bar{Y}b_{t+1} = (1 + i_t^*) \frac{n}{\beta} \psi_t - \frac{m}{\omega\sigma_s^2} (i_t - i_t^* - E_t\Delta s_{t+1}).$$

Since $(i_t^* - s_t) \bar{P}\bar{Y}b_{t+1}$ and $(i_t^*) \frac{n}{\beta} \psi_t$ are very small comparing to the multiplication of the first term, i.e., $\bar{P}\bar{Y}b_{t+1}$, and $\frac{n}{\beta} \psi_t$, these terms are dropped out. I hence obtain the approximation as

$$\bar{P}\bar{Y}b_{t+1} = \frac{n}{\beta} \psi_t + \frac{m}{\omega\sigma_s^2} (i_t^* - i_t + E_t\Delta s_{t+1}).$$

This leads to the uncovered parity condition as

$$i_t^* - i_t + E_t\Delta s_{t+1} = \chi_1 b_{t+1} - \chi_2 \psi_t,$$

where $\chi_1 = \frac{\bar{P}\bar{Y}}{m/(\omega\sigma_s^2)}$, $\chi_2 = \frac{n/\beta}{m/(\omega\sigma_s^2)}$.

APPENDIX C. CHAPTER 3 APPENDIX

C.1 The Real Stochastic Discount Factor or the Real Pricing Kernel

I begin to work with an approximation of the real pricing kernel. Taking logarithms in equation (3.9), I will have

$$m_{t+1} = \log(\beta) + (\rho - 1)x_{t+1} + (\alpha - \rho)(\log(W_{t+1}) - \log(\mu_t(W_{t+1}))),$$

where W_t is the value function. First, I will approximate the term $\log(W_{t+1}) - \log(\mu_t(W_{t+1}))$. Define the log wealth consumption ratio $wc_t \equiv \log W_t/c_t$. It can be written as

$$wc_t = \rho^{-1} \log[(1 - \beta) + \beta \exp(\rho g_t)],$$

where $g_t \equiv \log(\mu_t(\exp(wc_{t+1}) + x_{t+1}))$. Taking a linear approximation of the right-hand side as a function of g_t around the point \bar{m} , I get

$$\begin{aligned} wc_t &\approx \rho^{-1} \log[(1 - \beta) + \beta \exp(\rho \bar{m})] + \left[\frac{\beta \exp(\rho \bar{m})}{1 - \beta + \beta \exp(\rho \bar{m})} \right] (g_t - \bar{m}) \\ &\equiv \bar{k} + kg_t. \end{aligned}$$

Next, I conjecture a solution for the value function of the following form

$$wc_t = \bar{\omega} + \omega_z z_t + \omega_u u_t + \omega_w w_t.$$

Thus, $wc_{t+1} + x_{t+1} = \bar{\omega} + \omega_z z_{t+1} + \omega_u u_{t+1} + \omega_w w_{t+1} + x_{t+1}$.

Using the lognormality property, g_t can be expressed as $E_t(wc_{t+1} + x_{t+1}) + \frac{\alpha}{2} \text{var}_t(wc_{t+1} + x_{t+1})$.

Using this fact, then the log-wealth consumption ratio becomes

$$wc_t = \bar{k} + k[\psi + \frac{\alpha}{2}(\omega_u^2 \sigma_u^2 + \omega_w^2 \sigma_w^2)] + k(\omega_z \varphi_z + 1)z_t + k(\omega_u \varphi_u + \frac{\alpha}{2})u_t + k(\omega_w \varphi_w + \frac{\alpha}{2}\omega_z^2)w_t,$$

where $\psi = \bar{\omega} + \omega_u(1 - \varphi_u)\theta_u + \omega_w(1 - \varphi_w)\theta_w + \mu$.

I solve for the value function by matching coefficient with the conjecture form. I will have these coefficients:

$$\begin{aligned}\omega_z &= \frac{k}{1 - k\varphi_z} \\ \omega_u &= \frac{\alpha}{2} \frac{k}{1 - k\varphi_u} \\ \omega_w &= \frac{\alpha}{2} \omega_z^2 \frac{k}{1 - k\varphi_w}.\end{aligned}$$

Substituting and collectecting terms, $\log(W_{t+1}) - \log(\mu_t(W_{t+1}))$ becomes

$$\log(W_{t+1}) - \log(\mu_t(W_{t+1})) = \omega_z \sqrt{w_t} \varepsilon_{t+1}^z + \omega_u \sigma_u \varepsilon_{t+1}^u + \omega_w \sigma_w \varepsilon_{t+1}^w + \sqrt{u_t} \varepsilon_{t+1}^x - \frac{\alpha}{2} (\omega_z^2 w_t + \omega_u^2 \sigma_u^2 + \omega_w^2 \sigma_w^2 + u_t).$$

Finally, the real pricing kernel can be expressed as

$$-m_{t+1} = \delta^r + \gamma_z^r z_t + \gamma_u^r u_t + \gamma_w^R w_t + \lambda_x^r \sqrt{u_t} \varepsilon_{t+1}^x + \lambda_z^r \sqrt{w_t} \varepsilon_{t+1}^z + \lambda_u^r \sigma_u \varepsilon_{t+1}^u + \lambda_w^r \sigma_w \varepsilon_{t+1}^w,$$

where $\delta^r = [-\log(\beta) + (1 - \rho)\mu + \frac{\alpha}{2}(\alpha - \rho)]$, $\gamma_z^r = 1 - \rho$, $\gamma_u^r = \frac{\alpha}{2}(\alpha - \rho)$, $\gamma_w^r = \frac{\alpha}{2}(\alpha - \rho)\omega_z^2$, $\lambda_x^r = 1 - \alpha$, $\lambda_z^r = -(\alpha - \rho)\omega_z$, $\lambda_u^r = -(\alpha - \rho)\omega_u$, $\lambda_w^r = -(\alpha - \rho)\omega_w$.

C.2 The Coefficients

The coefficients of the conjecture solution for inflation can be shown as follows:

$$\begin{aligned}
a_1 &= \frac{(\tau_\pi^* - \varphi_z - \tau_s^*)(\gamma_z^r - \tau_z)}{(\tau_\pi - \varphi_z + \tau_s)(\tau_\pi^* - \varphi_z - \tau_s^*) + \tau_s \tau_s^*}, & a_1^* &= \frac{-\tau_s^*(\gamma_z^r - \tau_z)}{(\tau_\pi - \varphi_z + \tau_s)(\tau_\pi^* - \varphi_z - \tau_s^*) + \tau_s \tau_s^*} \\
a_2 &= \frac{\tau_s(\gamma_z^{*r} - \tau_z^*)}{(\tau_\pi - \varphi_z^* + \tau_s)(\tau_\pi^* - \varphi_z^* - \tau_s^*) + \tau_s \tau_s^*}, & a_2^* &= \frac{(\tau_\pi - \varphi_z^* + \tau_s)(\gamma_z^{*r} - \tau_z^*)}{(\tau_\pi - \varphi_z^* + \tau_s)(\tau_\pi^* - \varphi_z^* - \tau_s^*) + \tau_s \tau_s^*} \\
a_3 &= \frac{(\tau_\pi^* - \varphi_u - \tau_s^*)(\gamma_u^r - \frac{(\lambda_x^r)^2}{2})}{(\tau_\pi - \varphi_u + \tau_s)(\tau_\pi^* - \varphi_u - \tau_s^*) + \tau_s \tau_s^*}, & a_3^* &= \frac{-\tau_s^*(\gamma_u^r - \frac{(\lambda_x^r)^2}{2})}{(\tau_\pi - \varphi_u + \tau_s)(\tau_\pi^* - \varphi_u - \tau_s^*) + \tau_s \tau_s^*} \\
a_4 &= \frac{\tau_s(\gamma_u^{*r} - \frac{(\lambda_x^{*r})^2}{2})}{(\tau_\pi - \varphi_u^* + \tau_s)(\tau_\pi^* - \varphi_u^* - \tau_s^*) + \tau_s \tau_s^*}, & a_4^* &= \frac{(\tau_\pi - \varphi_u^* + \tau_s)(\gamma_u^{*r} - \frac{(\lambda_x^{*r})^2}{2})}{(\tau_\pi - \varphi_u^* + \tau_s)(\tau_\pi^* - \varphi_u^* - \tau_s^*) + \tau_s \tau_s^*} \\
a_5 &= \frac{(\tau_\pi^* - \varphi_w^* - \tau_s^*)(\gamma_w^r - \frac{(\lambda_z^r + a_1)^2}{2}) - \frac{\tau_s(a_1^*)^2}{2}}{(\tau_\pi - \varphi_w + \tau_s)(\tau_\pi^* - \varphi_w^* - \tau_s^*) + \tau_s \tau_s^*}, & a_5^* &= \frac{-\tau_s^*(\gamma_w^r - \frac{(\lambda_z^r + a_1)^2}{2}) - \frac{(\tau_\pi - \varphi_w + \tau_s)(a_1^*)^2}{2}}{(\tau_\pi - \varphi_w + \tau_s)(\tau_\pi^* - \varphi_w^* - \tau_s^*) + \tau_s \tau_s^*} \\
a_6 &= \frac{\frac{-(\tau_\pi^* - \varphi_w^* - \tau_s^*)a_2^2}{2} + \tau_s(\gamma_w^{*r} - \frac{(\lambda_z^{*r} + a_2^*)^2}{2})}{(\tau_\pi - \varphi_w^* + \tau_s)(\tau_\pi^* - \varphi_w^* - \tau_s^*) + \tau_s \tau_s^*}, & a_6^* &= \frac{(\tau_\pi - \varphi_w^* + \tau_s)(\gamma_w^{*r} - \frac{(\lambda_z^{*r} + a_2^*)^2}{2}) + \frac{\tau_s^*(a_2^*)^2}{2}}{(\tau_\pi - \varphi_w^* + \tau_s)(\tau_\pi^* - \varphi_w^* - \tau_s^*) + \tau_s \tau_s^*} \\
a_7 &= \frac{-\tau_s}{(\tau_\pi - \varphi_v + \tau_s)(\tau_\pi^* - \varphi_v - \tau_s^*) + \tau_s \tau_s^*}, & a_7^* &= \frac{\tau_s^*}{(\tau_\pi - \varphi_v + \tau_s)(\tau_\pi^* - \varphi_v - \tau_s^*) + \tau_s \tau_s^*} \\
a_8 &= \frac{-\tau_s}{(\tau_\pi - \varphi_v^* + \tau_s)(\tau_\pi^* - \varphi_v^* - \tau_s^*) + \tau_s \tau_s^*}, & a_8^* &= \frac{-(\tau_\pi - \varphi_v^* + \tau_s)}{(\tau_\pi - \varphi_v^* + \tau_s)(\tau_\pi^* - \varphi_v^* - \tau_s^*) + \tau_s \tau_s^*} \\
a_9 &= \frac{\frac{-1}{2}[(\tau_\pi^* - \varphi_e - \tau_s^*)(a_7)^2 + \tau_s(a_7^*)^2]}{(\tau_\pi - \varphi_e + \tau_s)(\tau_\pi^* - \varphi_e - \tau_s^*) + \tau_s \tau_s^*}, & a_9^* &= \frac{\frac{1}{2}[\tau_s^*(a_7)^2 - (\tau_\pi - \varphi_e + \tau_s)(a_7^*)^2]}{(\tau_\pi - \varphi_e + \tau_s)(\tau_\pi^* - \varphi_e - \tau_s^*) + \tau_s \tau_s^*} \\
a_{10} &= \frac{\frac{-1}{2}[(\tau_\pi^* - \varphi_e^* - \tau_s^*)(a_8)^2 + \tau_s(a_8^*)^2]}{(\tau_\pi - \varphi_e^* + \tau_s)(\tau_\pi^* - \varphi_e^* - \tau_s^*) + \tau_s \tau_s^*}, & a_{10}^* &= \frac{\frac{1}{2}[\tau_s^*(a_8)^2 - (\tau_\pi - \varphi_e^* + \tau_s)(a_8^*)^2]}{(\tau_\pi - \varphi_e^* + \tau_s)(\tau_\pi^* - \varphi_e^* - \tau_s^*) + \tau_s \tau_s^*}
\end{aligned}$$